

LoMoReVI meeting
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LoMoReVI beyond mathematical fuzzy logic

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Motivation

I suggest to invert 'our usual approach' to vagueness:

Instead of

trying to sell mathematical fuzzy logic as a model for reasoning under vagueness

I suggest to

analyze other theories and viewpoints about reasoning under vagueness first, and only then look for various possible applications of our tools and results (in that wider context).

Contents

- ▶ A reminder on the 'vagueness debate'
- ▶ Classification of ToVs (another reminder)
- ▶ Degrees of emphasis vs. degrees of truth
- ▶ Fuzzification vs. Recovering degrees
- ▶ Possible roles of FL within ToVs
(by way of brainstorming)

A reminder on the 'vagueness debate'

The literature on vagueness, even when restricted to logical foundations, is vast and goes back to antiquity. (More than 500 books and papers, till 2003, in Justin Needle's bibliography.)

There is wide discrepancy about what exactly vagueness is:

- ▶ Linguistic? ontological? epistemic? metaphysical?
- ▶ Not always distinguished from: indeterminacy, unspecificity, excessive generality, uncertainty
- ▶ However: a canon of central features is to be addressed: Sorites paradox, borderline cases, higher order vagueness, . . .

Frequently, the debate goes meta-level nowadays:

- ▶ What counts as a phenomenon of vagueness?
- ▶ Which role does formal logic play?
- ▶ What criteria does a theory of vagueness have to satisfy?
- ▶ How to classify theories of vagueness?

A possible classification of ToVs

NB: all ToVs acknowledge vagueness as ubiquitous and useful
and all offer their own to Sorites paradoxa

Gap theories:

- ▶ vagueness leaves **truth value gaps**
- ▶ **classical logic** is defended in principle, but Kleene-type **3-valued logic** is frequently used in practice
- ▶ **Proponents:** (Frege, Russel, Peirce), Tye, Blau, ...

Epistemic theories:

- ▶ **ontologically** boundaries are **sharp**, but we **cannot recognize** corresponding cut-off points (for principal reasons)
- ▶ **classical logic** is defended, but **modal operators** are used to express 'clearly', 'indefinitely', etc.
- ▶ **Proponents:** (Chrysippus), Williamson, Sorensen, Cargile, ...
Also: J. Halpern(!) – his central notion: indistinguishability

Supervaluationism:

- ▶ semantics of vague expressions to be specified against a background **space of admissible precisifications**
- ▶ slogan: 'truth is **supertruth**'
- ▶ **Proponents:** Fine, Varzi, Keefe, ...

Pragmatism:

- ▶ vagueness indicates **insufficient specification** of a language
- ▶ subject: **choice** of appropriate (classical) language
- ▶ **Proponents:** Lewis, Graff, Burns, ...

Currently widely discussed:

Contextualism:

- ▶ semantics of vague expressions always momentarily fixed with respect to a 'conversational score' (context)
- ▶ key concepts: open texture, principle of tolerance
- ▶ corresponding logic shares features of intuitionistic logic and supervaluation, but extends to appropriate modal operators
- ▶ Proponents: Shapiro, Raffman, Soames, Kamp, Gaifmann, . . .

Most relevant for Fuzzy Logic:

Degree based theories

- ▶ slogan: 'truth comes in degrees'
- ▶ adequateness of truth functionality not necessarily implied
- ▶ different accounts of how degrees arise
- ▶ different derivations of connectives from first principles
- ▶ Proponents (outside FL): Machina, Edgington, Wright, ...
N.J.J. Smith: 'Vagueness and Degrees of Truth', OUP, January 2009)

Fuzzy Logic (partly) seems to fit, but is it a 'theory of vagueness'?

Main worries:

- ▶ How to justify particular truth values and functions?
- ▶ What does it mean to say that FL is 'only a model'?

A Side Remark: Degrees of emphasis vs. degrees of truth

Identifying truth values with degrees of emphasis might connect pragmatism (emphasis) with fuzzy logic (graded truth).

However, there are many problems:

- ▶ linguists have studied emphasis a lot, but don't see any connection with truth degrees¹
- ▶ degrees of emphasis are used to convey degrees of importance and salience, maybe also degrees of belief (but hardly of truth)
- ▶ emphasis is mostly applied to determinate (crisp) statements
- ▶ are degrees of emphasis propagated compositionally with respect to logical connectives?
- ▶ even if yes: is it really an issue of vagueness?
Does shouting, e.g., 'Europe is great' really imply that the speaker does not recognize its vagueness?

¹Even if you don't trust experts, remember that they are at least as reliable and informative than just a collection of 'intuitions'!

Fuzzification vs. Recovering degrees

N.B. Every semantic predicate can be 'fuzzified' ('... leads to a truth gap', '... is not known', '... is not distinguishable from ...', '... is in the conversational score', '... is an admissible precisification', '... is the right target language', '... entails ...', '... is true', etc.) but the **crucial questions** are:

- ▶ Does this fuzzification deepen the corresponding analysis?
- ▶ Are the degrees imposed onto or recovered from the model?

Positive examples:

- ▶ Giles's game for Lukasiewicz logic:
truth values **emerge** from the dispersiveness of repeatable 'atomic experiments' – truth functionality **is derived** from appropriate dialogue game rules
- ▶ Esteva/Godo/Hajek-approach to modelling the semantics of the modal operator 'probably' (or 'I believe that ...')

Possible roles of FL – an exercise in brainstorming

Gap theories:

- ▶ are intermediate truth values a convincing generalization of truth value gaps?
- ▶ gaps as 'undefinedness': definedness may depend on context and therefore, globally, be a matter of degree

Epistemic theories:

- ▶ truth degrees may be recovered from (maximal) iterations of (e.g., Williamson's) modality '(in)determinately'
- ▶ degrees of distinguishability

Supervaluationism:

- ▶ recovering degrees from the ratio of positive to negative (admissible) precisifications
- ▶ 'building in' degrees of admissibility in precisification space
→ fuzzy sets of 'admissible precisifications'
- ▶ admissibility as a fuzzy relation (equivalent to degrees of admissibility?)

Pragmatism:

- ▶ do FLs naturally arise from stipulating that the set of (crisp, interpreted) target languages is fuzzy?

Contextualism:

- ▶ can fuzzy truth values be used to 'abstract away' from specific contexts? (local vs. global assertibility)
- ▶ models using 'fuzzy contexts' – e.g., entries in the relevant conversational score (D. Lewis) may disappear gradually with advancing time

Degree based theories:

- ▶ N.B. not all degree based ToVs lead to fuzzy logic!
- ▶ E.g., the role of truth functionality (or truth functional fragments of other logics) remains to be investigated

Conclusions

- ▶ Different approaches to vagueness may help to understand different aspects of vagueness
- ▶ Different application scenarios may call for different background theories
- ▶ Not only is a ToV more than a family of formal logics, but also: a LoMoReVI should consist in more, too
- ▶ Mathematical fuzzy logic can in many ways become a component of a full-fledged LoMoReVI that goes beyond a particular way to formalize 'preservation of truth degrees'!