A principled approach to fuzzy rule base interpolation using similarity relations

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Abstract

In this work we consider a very general principle for fuzzy rule interpolation methods based on an interpretation of the generalized modus ponens rule in terms of closeness relations. Then we present two particular instances of the general principle when the closeness relations are defined from parametric families of similarity fuzzy relations on the input and output spaces. The case of multiple input variables is also considered.

Keywords: fuzzy rules, similarity relations, interpolation, implication measures

1 Introduction

Ideally, for any input, a fuzzy rule based system should be able to produce a useful output (neither the empty set nor the output variable universe of discourse). To do so, every input value should be covered by at least one rule in the base and the outputs produced by the rules as a whole should not be completely contradictory.

On the other hand, the size of fuzzy rule bases usually grow exponentially as the number of input variables increase. When the number of input variables is large, this often leads to the construction of sparse rule bases, that demands either lesser efforts, when the rules are elicited from human experts, or less data, when the rules are extracted automatically by a learning algorithm. But, the sparser the rules base, the greater are the chances that useful outputs for any input cannot be guaranteed [1].

The generalized Modus Ponens is the basis of fuzzy rule bases inference machinery. Given a fuzzy rule "If X is A_i then Y is B_i", it can be stated as the (high level) rule

"The more the input A is *close to* A_i
the more the output B must be *close to* B_i."

Given a rule base $K = \{$"If X is A_i then Y is B_i" $\}_{i\in I}$ and an input "X is A", in the usual implementation of this rule, the first step consists in verifying how much input A is similar or included in each A_i, that stands for the implementation of concept of "closeness" in relation to the input variable. Then using this degree of similarity or inclusion, the B_i's are modified, thus defining the concept of "closeness" in relation to the output variable. Then these modified terms are aggregated and, in some applications, a precise value is chosen for the output variable based on this aggregated result. Unfortunately, when A is not covered, this scheme does not produce any useful result.

Interpolation of rules is a reasonable solution for the covering problem, if the underlying knowledge allows for a smooth transition from one rule to the other. Several approaches can be found in the literature to find outputs from a systems of sparce rules. This type of problem is studied firstly in [11, 12] where the method proposed is based on preservation of the proportions in the universes of inputs an outputs. In [14] the authors proved that even all fuzzy sets involved are normalized and convex triangular fuzzy sets, the solution ob-

[1]Note that also in the case of non-sparse rule bases, the covering problem may also appear. Namely, when there is no rule that covers all the input variables values at the same time.

tained by Koczy-Hirota method may not be a fuzzy set and they propose some modification of the method to avoid this problem. Other methods have been proposed in [13], [5] and [3]. In particular the method presented in [3] is based on the preservation of a similarity measure between fuzzy sets, in [7] the authors extend the original system by new rules obtained by convex combination and in [4] a comparative study of the results of different methods proposed are given. Finally in [8] a new method based on Ruspini's implication measure is presented and shown to be closely related to [3] in some particular case.

In this work, we consider a very general principle for interpolation methods based on a similarity (inclusion)-based interpretation of the generalized MP rule above. This is done in Section 2. In Section 3 we present an approach as two particular instances of the general principle considering a single input variable, while Section 4 details an application of the method. Section 5 addresses the case of multiple input variables and Section 6 discusses how, for a given an input, to select a subset of the fuzzy rule base to perform interpolation. Finally, Section 7 brings the conclusions.

2 General principles

Assume a relationship between the metric notions of closeness at work in the input and output spaces is given. To start with assume further that this is given by a pair of graded closeness relations on fuzzy sets $close_X$ and $close_Y$. In this way we are not only able to compare in one space whether one fuzzy set A is closer to A' than to another A'' (when $close_X(A, A') \geq close_X(A, A'')$), but also to perform comparisons across the two spaces, e.g. B and B' are at least as close as A and A' (when $close_X(A, A') \geq close_X(B, B')$). Then the idea is to interpret a fuzzy rule "If $X = A_i$ then $Y = B_i$" as a kind of meta-inference rule stating "The *closer* the input A is to A_i, the *closer* the ouput B is to B_i" as

$$close_X(A, A_i) \leq close_Y(B, B_i), \quad \text{(C1)}$$

Now, if we have a set of rules $K = \{$ if $X = A_i$ then $Y = B_i\}_{i \in I}$, we will say that K allows a similarity-based interpolation inference with respect to the metric (in a very general sense) pair $(close_X, close_Y)$ if condition (C1) is satisfied by all rules in K, in the sense that the following set of conditions

$$\{close_X(A_j, A_i) \leq close_Y(B_j, B_i) \mid i, j \in I\}, \quad \text{(C2)}$$

hold. If so, then given an input $X = A$, a compatible output $Y = B$ according to K and $(close_X, close_Y)$ would be among those which are solutions of the following inequalities

$$\{\alpha_i \leq close_Y(B, B_i) \mid i \in I\}, \quad \text{(C3)}$$

where $\alpha_i = close_X(A, A_i)$.

If we define $close_X(A, A') = \inf_x A(x) \to_* A'(x)$ and $close_Y(B, B') = \inf_y B(y) \to_* B'(y)$, where $\to_*$ is the residuum of a left-continuous t-norm, then the inference model which takes as output the least specific solution of (C3) is Boixader and Jacas' extensionalty based model of approximate reasoning [2].

Boixader and Jacas' s is a good model as soon as $\inf_x A(x) \to_* A'(x) > 0$, otherwise we cannot infer anything. This is usually the case when the fuzzy rule base is sparse and the closeness relations used are somehow based on some notion of fuzzy set inclusion degree. Therefore we need closeness relations not so strict which provide positive degrees even for totally disjoint fuzzy sets. This leads us to consider a richer framework where the relationship between the metric notions of closeness at work in the input and output spaces is not given by one but by a parametric family of pairs $\mathcal{M} = \{(close^X_\lambda, close^Y_\lambda)\}_{\lambda \in \Lambda}$, with $\Lambda \subset \mathbb{R}$ of closeness relations on fuzzy sets, in such a way that if $\lambda \leq \lambda'$ then $close^X_\lambda \leq close^X_{\lambda'}$ and $close^Y_\lambda \leq close^Y_{\lambda'}$. Moreover, we require that closeness relations for small values of λ tend to the equality relation and for larger values tend to the totally unrestricted relation (i.e. identically equal to 1). The aim is to have a richer representations of the notions of closeness, so that varying the parameter λ amounts to consider from stricter to wider representations.

Then, extending the above considerations to this framework, given a set of rules $K = \{$ if $X = A_i$ then $Y = B_i\}_{i \in I}$, we will say that K allows a similarity-based interpolation inference with respect to the metrics $\mathcal{M} = \{(close^X_\lambda, close^Y_\lambda)\}_{\lambda\in\Lambda}$ if condition (C1) is satisfied by all rules in K for all λ's, in the sense that the following set of conditions

$$\{close^X_\lambda(A_j, A_i) \leq close^Y_\lambda(B_j, B_i) \mid i, j \in I\}, \quad \text{(C4)}$$

hold for all λ. In such conditions, then a compatible output $Y = B$ for an input $X = A$, according to K and to the metrics $\{(close^X_\lambda, close^Y_\lambda)\}_{\lambda\in\Lambda}$ would be among those which are solutions of the following inequalities

$$\{\alpha_{\lambda,i} \leq close^Y_\lambda(B, B_i) \mid i \in I, \lambda \in \Lambda\}, \quad \text{(C5)}$$

where $\alpha_{\lambda,i} = close^X_\lambda(A, A_i)$.

The method described in [8] can be cast, in part, within this approach. In fact they use $close^X_\lambda, close^Y_\lambda$ defined by Ruspini's implication measures (see next section) induced by some parametric families of similarity relations on the input and output spaces, and the output B is required to satisfy (C5) with equalities. This stronger condition results in fact sometimes too strong to get a fuzzy set as solution.

Actually, given a fuzzy rule base RB, it is not usual to take the full RB into account to perform interpolation given an input $X = A$, but only a subset $K(A)$ of rules from RB which are most closely related to A.

The determination of $K(A)$ in itself is not trivial when RB is sparse and the rules have several input variables. Having $K(A)$ either too small or too large may lead to unsatisfactory results.

3 Proposed Approach: the case of single input variable

In the following, we detail an instance (in fact two) of the general approach which make use of fuzzy similarity relations to define the closeness relations $close^X_\lambda$ and $close^Y_\lambda$.

Suppose the universes of inputs and outputs are the same, the real line $\mathbb{R}$ for simplicity, and we have a parametric nested family of fuzzy similarity relations on $\mathbb{R}$, $\mathcal{S} = \{S_\lambda : 0 \leq \lambda \leq +\infty\}$, such that S_0 is the crisp equality and $S_{+\infty} = 1$.

We consider two families of closeness relations induced by $\mathcal{S}$. The first family is defined as the Ruspini's implication measures

$$close^1_\lambda(E, D) = \\ I_{S_\lambda}(D \mid E) = \inf_{u\in\mathbb{R}}\{E(u) \rightarrow_* (S_\lambda \circ D)(u)\}$$

where $\rightarrow_*$ is the residuum of some left-continuous t-norm $*$ and $\circ$ denotes $\max - \min$ composition. The second one is defined by symmetrization:

$$close^2_\lambda(E, D) = \min(I_{S_\lambda}(D \mid E), I_{S_\lambda}(E \mid D))$$

Notice that $close^1_\lambda$ is reflexive and $*$-transitive when so is S_λ, whereas $close^2_\lambda$ is in fact a $*$-similarity relation on fuzzy sets, i.e. it is reflexive and symmetric, and $*$-transitive again when so is S_λ. In the following we will write $close_\lambda$ when we do not distinguish between $close^1_\lambda$ and $close^2_\lambda$.

Let $RB = \{R_i$: if $X = A_i$ then $Y = B_i\}_{i\in I}$ be a fuzzy rule base, let $X = A$ be an input and let $K(A)$ be the subset of rules of RB most closely related to the input. Since $close_\lambda(E, D) \leq close(E, D')$ when $D \leq D'$, it makes sense the following definition. For each $\lambda \in \Lambda = [0, +\infty]$, we define

$$f(\lambda) = \inf\{\mu \mid \forall R_i, R_j \in K(A), \\ close_\lambda(A_i, A_j) \leq close_\mu(B_i, B_j)\}.$$

Actually, it makes only sense to consider the values of λ for which $f(\lambda) > 0$. Let $\lambda_0 = \inf\{\lambda \mid f(\lambda) > 0\}$. Then it is easy to check that $K(A)$ allows a similarity-based interpolation inference with respect to the metric family $\mathcal{M} = \{(close_\lambda, close_{f(\lambda)})\}_{\lambda\in[\lambda_0,+\infty]}$, namely $\mathcal{M}$ satisfies condition (C4).

In this setting the proposed approach consists of taking as output

$$B = \cap\{A \circ R^*_i \mid R_i \in K(A)\}$$

with

$$A \circ R^*_i = \cap\{A \otimes R^*_i(\lambda) \mid S_\lambda \in \mathcal{S}\},$$

where $A \otimes R^*_i(\lambda)$ is taken as the least-specific B' such that $close_\lambda(A, A_i) \leq close_{f(\lambda)}(B', B_i)$.

With both types of closeness relations it can be proved that $A \otimes R^*_i(\lambda)$ is given by $close_\lambda(A, A_i) \rightarrow_* (S_{f(\lambda)} \circ B_i)(y)$.

Proposition 1 $A \otimes R_i^*(\lambda) = close_\lambda(A, A_i) \rightarrow (S_{f(\lambda)} \circ B_i)(y)$.

Proof: Let $\alpha = close_\lambda(A, A_i)$. Let us first consider the case $close_{f(\lambda)}(B', B_i) = I_{S_\lambda}(B_i \mid B')$. Then, assume $\alpha \leq I_{f(x)}(B_i \mid B')$.

By definition, $\alpha \leq I_{f(x)}(B_i \mid B')$ iff $\alpha \leq B'(v) \rightarrow_* (S_{f(x)} \circ B_i)(v)$ for all v, hence iff $B'(v) \leq \alpha \rightarrow_* (S_{f(x)} \circ B_i)(v)$. Therefore the least B' which satisfies $\alpha \leq I_{f(x)}(B_i \mid B')$ is actually $B' = \alpha \rightarrow_* (S_{f(x)} \circ B_i)$.

Finally let us check that, in the case of the symmetric measure, i.e. when $close_{f(\lambda)}(B', B_i) = \min(I_{S_\lambda}(B_i \mid B), (I_{S_\lambda}(B \mid B_i)$, this B' also satisfies the second constraint $\alpha \leq I_{f(x)}(B' \mid B_i)$. In fact we have $I_{f(x)}(B' \mid B_i) = \inf_v B_i(v) \rightarrow_* (S_{f(x)} \circ B')(v) \geq \inf_v B_i(v) \rightarrow_* B'(v) = \inf_v B_i(v) \rightarrow_* (\alpha \rightarrow_* (S_{f(x)} \circ B_i)(v)) = \inf_v \alpha \rightarrow_* (B_i(v) \rightarrow (S_{f(x)} \circ B_i)(v)) = \alpha \rightarrow_* 1 = 1$. □

The interpolated solution for an input $X = A$ would then be the following *big* intersection:

$$Interpol_{RB,\mathcal{M}}(A) = A \circ^* K(A) = \bigcap_{R_i \in K(A)} \bigcap_{\lambda \geq 0} close_\lambda(A, A_i) \rightarrow_* (S_{f(\lambda)} \circ B_i).$$

Observe that even though the formal expression for the ouput is the same for both types of closeness relations $close^1_\lambda$ and $close^2_\lambda$, the fuzzy set result is not the same. Indeed, both the values $close_\lambda(A_i, A)$ and the function f depend on the particular definition of the relations $close_\lambda$.

4 Example

Here we detail both approaches in the particular case of a crisp input A and a rule base with two rules $RB = \{R_i$: If X is A_i then Y is $B_i\}_{i=1,2}$ such that all A_i's, B_j's and input A are triangular fuzzy sets. A triangular fuzzy set will be described by a 3-tuple $[a, b, c]$ where $(-\infty, a) \cup (c, \infty)$ is the complement of the support and $\{b\}$ is the core. On the other hand the parametric family of fuzzy similarity relations on $\mathbb{R}^2$ is defined by

$$S_\lambda(x, y) = \max(1 - \frac{|x - y|}{\lambda}, 0)$$

for any real $\lambda > 0$, and $S_0(x, y)$ is the classical equality. Notice that $S_\lambda \circ [a, b, c] = [a - \lambda, b, c + \lambda]$.

Finally in the definition of implication measures we use Gödel implication. In such a case, letting $W_\lambda = \{x \mid E(x) > (S_\lambda \circ D)(x)\}$, we actually have

$$I_\lambda(D \mid E) = \begin{cases} \inf(W_\lambda), & \text{if } W_\lambda \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

To further simplify the computations, we can assume, without loss of generality, that $Core(A_1) = Core(B_1) = \{0\}$ and $Core(A_2) = Core(B_2) = \{p\}$, with $p > 0$. This amounts to normalize the domains of X and Y. Indeed we consider the following description of the fuzzy sets involved:

$A_1 = [-h, 0, l]$, $B_1 = [-h', 0, l']$,
$A_2 = [p - q, p, p + k]$, $B_2 = [p - q', p, p + k']$,

where $h, l, h', l', q, k, q', k' \geq 0$. We also assume that $l < p - q$ and $l' < p - q'$, that is, rules are considered to be sparse.

We now instantiate the method described in the previous section to the current setting when "close to" is modeled by the relations $close^2_\lambda$. Consider rules R_1 and R_2 and let us compute $f(\lambda)$ (first we need to compute $close^2_\lambda$):

- if $q + k \geq h + l$ then $close^2_\lambda(A_1, A_2) = I_\lambda(A_1 \mid A_2)$.

  if $q' + k' \geq h' + l'$ then $close^2_\lambda(B_1, B_2) = I_\lambda(B_1 \mid B_2)$.

- if $q + k \leq h + l$ then $close^2_\lambda(A_1, A_2) = I_\lambda(A_2 \mid A_1)$.

  if $q' + k' \leq h' + l'$ then $close^2_\lambda(B_1, B_2) = I_\lambda(B_2 \mid B_1)$.

- we let $f(\lambda)$ undefined for $\lambda < r_0 = \max(p - q + h, p + k - l)$.

- Let $r'_0 = \max(p - q' + h', p + k' - l')$. Then for $\lambda \geq r_0$, $f(\lambda) = \lambda + (r'_0 - r_0)$.

Figure 1 shows the values of $\alpha = I_\lambda(A_1 \mid A_2), r_0, r'_0, \lambda$ and $f(\lambda)$ for a given configuration of rules that corresponds to the first case of the last list of cases.

Actually in Figure 2 we compute the output of the system for a precise input $X = a$ for our rule base RB depicted in Figure 1. The method to compute the output is as follows. First we consider rule

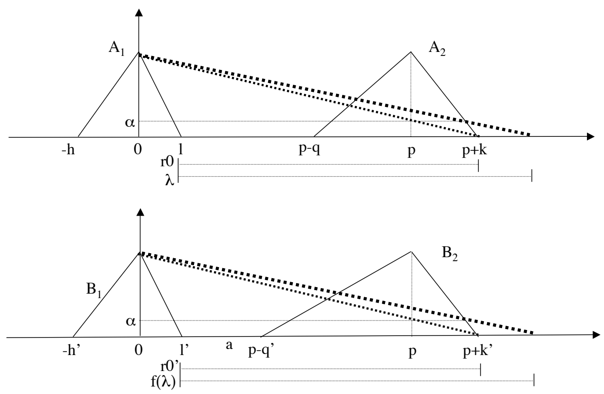

Figure 1: Illustration of $f(\lambda)$ calculation.

R_1 and compute $close^2_\lambda(a, A_1) = I_\lambda(A_1 \mid a) \leq I_\lambda(a \mid A_1)$, which, for the particular value of λ given in Figure 1, is β. For this first rule R_1 we compute the Fuzzy set $B(\lambda)$ for each λ (Two of the corresponding results $B(\lambda)$ for values λ and λ' are explicitely given in fig.2). Then we aggregate the results obtaining the fuzzy set B'. Moreover it is necessary to repeat the proces for rule R_2 and similarly we obtain the fuzzy set B''. The proposed output is $B = B' \cap B''$.

The non-symmetric case is obtained when we model "close to" by the relations $close^1_\lambda$. The main differences with the symmetric case are the following:

- To compute function f, one has to consider two inequalities: $I_\lambda(A_1 \mid A_2) \leq I_\lambda(B_1 \mid B_2)$ and $I_\lambda(A_2 \mid A_1) \leq I_\lambda(B_2 \mid B_1)$.
- Then, we have to compute $B' = \cap_\lambda B'(\lambda)$ and $B'' = \cap_\lambda B''(\lambda)$ in the same way as in the previous case with the differences due to the use of $close^1_\lambda$ instead of $close^2_\lambda$.

As mentioned previously, this non-symmetric approach is very similar to the one presented in [8] when only implication measures are used, actually the computation of $f(\lambda)$ is slightly different.

5 Proposed method - multiple input variables

The approach presented in the previous sections can be easily extended to the multiple input variables case. Let

$$K = \{R_i : \text{If } X_1 \text{ is } A_{1i} \text{ and ... and } X_m \text{ is } A_{mi} \text{ then } Y \text{ is } B_i\}_{i=1,n}$$

be a fuzzy rule base and

$$A :< A_1, ..., A_m >$$

be the multidimensional input. Once $K(A)$ is determined, we define

$$f(\lambda_1, ..., \lambda_m) = \inf\{\lambda \mid \forall R_i, R_j \in K(A), \alpha_i \leq close_\lambda(B_i, B_j)\},$$

where $\alpha_i = min_{j=1,m} close_{\lambda_j}(A_1, A_{1j})$. Then for the two kinds of $close_\lambda$ relations, the interpolated solution for an input would then be

$$Interpol_K(A_1, ..., A_m) =< A_1, ..., A_m > \circ^* K = \bigcap_{R_i \in K(A)} \bigcap_{\lambda \geq 0} \alpha_i \rightarrow (S_{f(\lambda_1,...,\lambda_m)} \circ B_i)(y).$$

Note that Proposition 1 does not depend on the original value α_i and thus we obtain the same proof.

6 Determination of K(A)

Let us suppose we have a rule base $K = \{R_i$: If X_1 is $A_{1,1_i}$ and ... and X_m is A_{m,m_i} then Y is $B_{y_i}\}_{i=1,n}$ and an input $A :< A_1, ..., A_m >$.

Let us also suppose our framework is such that

- All fuzzy terms in the rules premises and the terms constituting A are fuzzy sets with compact support.
- All fuzzy terms are given in $\mathbb{R}$ and can be ordered, e.g. $D_1, ..., D_k$, and are such that $\forall i, \forall x \in \mathbb{R}, D_i(x) + D_{i+1}(x) \leq 1$.
- Every one of the terms of a given input variable has at least one rule that addresses it.

Let us suppose further that input A is not covered by any rule in K, i.e.,

$$\forall (x_1, \ldots, x_m) \in \mathbb{R}^m, \forall i \in \{1, .., n\}, \exists k \in \{1, ..., m\}, \min(A_k(x_k), A_{k,k_i}(x_k)) = 0.$$

We are interested to construct a set $K(A)$, containing the rules whose input terms are the closest to input A, according a to given metric. We propose to create $K(A)$ such that a rule R_i in K will

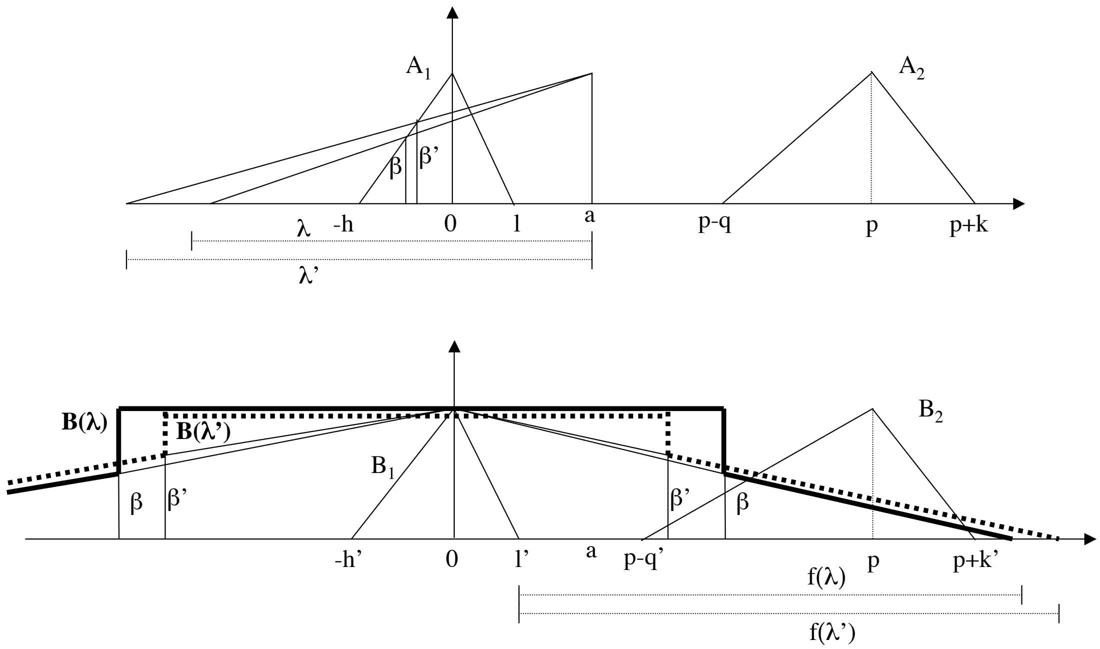

Figure 2: Illustration of output determination.

belong to $K(A)$ either when it has the minimal distance to A or when R_i is closer to input A itself than to the rules already in the close vicinity of A.

Formally, we construct $K(A)$ iteratively in the following manner. Let $K(A)^l$ denote set $K(A)$ obtained at the l-th step and let $dmin = \inf_{R_j \in K} d(R_j, A)$ denote the minimum distance of a rule in K to the input A, according to a distance d to be defined later. We initially make $K(A)^0 = \{R_j \mid d(R_j, A) = dmin\}$. Subsequently, a rule R_i will belong to $K(A)^l, l > 0$ if $R_i \in K(A)^{l-1}$ or $d(R_i, A) \leq \inf\{d(R_i, R_j) \mid R_j \in K(A)^{l-1}\}$. The algorithm stops when we obtain $K(A)^l = K(A)^{l+1}$.

To define distance d we will first index each rule with the sequence of indices corresponding to its terms; a rule "If X_1 is $A_{1,1_i}$ and ... and X_m is A_{m,m_i} then Y is B_{y_i}" will be denoted as $R_{1_i,\ldots m_i,y_i}$. For instance, $R_{34,2}$ denotes rule "If X_1 is $A_{1,1_3}$ and X_2 is $A_{2,2_4}$ then Y is B_2", where A_{r_s} stands for the s-th term of variable X_r.

The distance between the input of any two rules $R_{1_i,\ldots,m_i,y_i}$ and $R_{1_j,\ldots,m_j,y_j}$ is given by $d(R_{1_i,\ldots,m_i,y_i}, R_{1_j,\ldots,m_j,y_j}) = \Sigma_{k=1,m} \; |k_i - k_j|$. For instance, d$(R_{21,1}, R_{43,2}) = |2-4| + |1-3| = 4$. In the following we omit the output term index to simplify notation.

The distance between the input of a rule and the input itself will be given by the distance of the rule to any of the (hypothetical) rules in $H(A)$, that consists of the rules in the closest possible hypercube of rules surrounding the input. For a rule base with only 2 input variables, the distance between a rule $R_{1_i 2_i}$ and input A is given by $d(R_{1_i 2_i}, A) = inf_{R_{uv} \,\in\, H(A)} d(R_{1_i 2_i}, R_{uv})$. If input $A =< A_1, A_2 >$ is such that A_1 lies between $A_{1,1_s}$ and $A_{1,1_{s+1}}$ and A_2 lies between terms $A_{2,2_r}$ and $A_{2,2_{r+1}}$, then $H(A) = \{R_{sr}, R_{s(r+1)}, R_{(s+1)r}, R_{(s+1)(r+1)}\}$.

Let us suppose A is such that A_1 lies between $A_{1,1}$ and $A_{1,2}$ and A_2 lies between terms $A_{2,1}$ and $A_{2,2}$. In consequence, $H(A) = \{R_{11}, R_{12}, R_{21}, R_{22}\}$. Let us suppose we have $RB = \{R_{13}, R_{22}, R_{31}\}$. In this case, $K(A)^0 = \{R_{22}\}$ and, since the input terms in R_{13} and R_{31} are closer to the input terms of the minimal-distance rule R_{22} than to input A itself, we finally have $K(A) = \{R_{22}\}$. On the other hand, if $RB = \{R_{11}, R_{23}, R_{32}, R_{33}\}$, we have $K(A) = \{R_{11}, R_{23}, R_{32}\}$: R_{11} belongs to $K(A)$ because it is the rule with the minimal distance to A ($K(A)^0 = \{R_{11}\}$), R_{23} and R_{32} both belong to $K(A)$ because they are closer to A than to R_{11}, whereas R_{33} does not belong to $K(A)$ because it is closer to R_{23} and R_{32} than to A.

7 Conclusions

Given a sparse fuzzy rule base, in this work we have considered a general principle for fuzzy rule interpolation methods by establishing a suitable correspondence between families of closeness relations on fuzzy sets from the input and output spaces. This general approach, which encompasses some approximate reasoning models in the literature, has been particularized when the closeness relations are defined by means of parametric families of similarity relations on the input and output domains. The comparison with other interpolation methods proposed and the both theoretical and empirical behavior of the proposed approach remains as the next task to work on.

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