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MANY-VALUED EPISTEMIC STATES. AN APPLICATION TO A REFLECTIVE ARCHITECTURE: MILORD-II*

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ABSTRACT

Halpern and Moses [Halpern & Moses, 84] define and characterize what a minimal epistemic state associated to a set of premises is, using the notions of stable set and S5-Kripke models. Based on such epistemic states, Halpern and Moses define an entailment relation with which one can infer what is known and, more importantly, what is unknown by an agent. In this paper we formulate an extension of these notions when the underlying logic is many-valued in order to capture knowledge which can be possibly pervaded with fuzziness. As an application example we study the logical foundation of a core fragment of the *MILORD II* architecture [Sierra & Godo, 92,93], focusing in particular on giving a modal interpretation of *MILORD II* meta-predicates. In this sense, this paper is mainly a contribution in studying the relationship between special meta-predicates in meta-level architectures for non-monotonic reasoning, such as *MILORD II* or *BMS* ([Tan & Treur, 91], [Tan, 92]), and modal operators in non-monotonic epistemic logics.

1. Introduction

The knowledge states of a rational and introspective agent are usually modelled as *stable sets* of epistemic formulas. Epistemic formulas are formulas in a language with a standard pair of epistemic (modal) operators standing for knowledge and possibility. In [Halpern & Moses, 84], Halpern and Moses define and characterize what a *minimal epistemic state* associated to a set of premises is, using the notions of stable set and S5-Kripke models. Based on such epistemic states, Halpern and Moses define an entailment relation with which one can infer what is known and, more importantly, what is unknown by an agent. This entailment relation is obviously non-monotonic, and provides the link of this epistemic theory to logical *meta-level architectures*. Namely, the entailment relation can be used to derive *meta-knowledge* about what is known and what is not from object-level formulas represented as non-modal formulas. Moreover, in [Meyer & van der Hoek, 93a, 93b], a default logic based on epistemic notions is introduced where the above mentioned meta-knowledge is used as input to derive default beliefs.

This formalism has been used up to now in a classical two-valued framework. However, many times we want to model agent states coping with *fuzzy knowledge*, in the sense that an agent's knowledge can incorporate propositions which can be assigned partial degrees of truth, and thus one is led to a many-valued calculus. To do this, an extension of the formalism is necessary. First of all, we extend the epistemic framework to be defined on top of a many-valued logic. Then we apply this extension to a meta-level architecture *MILORD II*. This is an architecture for Knowledge Based Systems (KBS) that combines reflection and modularization techniques, together with an approximate reasoning component based on many-valued logics. In this way, the system is able to deal with complex reasoning patterns in the large. In this paper we will investigate the logical foundation of a core fragment of *MILORD II*, focusing in particular on giving a modal interpretation of the *MILORD II* meta-predicate *WK* through the notion of Halpern & Moses' epistemic states, extended to the many-valued case.

2. The Modal Many-Valued Logic *MVEL*

In this section we introduce a particular modal logic *MVEL*, standing for *Many-valued Epistemic Logic*, based on a particular many-valued semantics, in the sense that propositional variables are interpreted in an arbitrary (finite) set A_n of n truth-values. The set of truth-values is taken as a scale of partial degrees of truth, ranging from *False* to *True*. However, as we will see, a special kind of unary connectives, known as *indicators* in the literature of many-valued logics, will make possible to always evaluate any formula to either *True* or *False*. This modal logic will serve us as the logical framework where to put into relation *MILORD II* reasoning system and Halpern and Moses' epistemic states.

From now on, we will take a generic chain of n

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elements

$$0 = a_1 < a_2 < ... < a_n = 1$$

where 0 and 1 are the booleans *False* and *True* respectively as the set A_n of truth-values on top of which formulas of the *MVEL language* will be valued. We begin with a description of the syntax and semantics of the *MVEL* language.

Formulas:
Given a finite set of atomic symbols At, formulas of the languague are built upon the set $\{(z)p \mid z \in A_n,\ p \in At\}$, which elements are called *quasi-atoms,* in the usual way with connectives $\wedge$, $\vee$, $\rightarrow$ and $\neg$, a modal operators $\Box$ denoting "it is known that", and its dual operator $\Diamond$:

- every quasi-atom is a formula
- if φ is a formula, so is $\neg\varphi$
- if φ and ψ are formulas so are $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$
- if φ is a formula, so are $\Box\varphi$ and $\Diamond\varphi$
- nothing else is a formula

Formulas built in this way will be referred as *MVEL-formulas*. Non-modal formulas will be also called *objective*.

Semantics:
A *MVEL* Kripke model is a structure $K = \langle W, \models, R \rangle$, where W is a set of possible worlds, $\models: At \times W \rightarrow A_n$ is, for each world, a valuation mapping of atoms into the set of truth-values A_n, and $R = W \times W$ is the universal accessibility relation. We will write $w(A) = z$ for $\models(A, w) = z$. Within such a model K, we will use the same symbol $\models$ to denote its induced interpretation of *MVEL*-formulas, i.e. the mapping

$$\models: MVEL\text{-}Formulas \times W \rightarrow \{0, 1\}$$

defined as follows (we use the expression $w \models \varphi$ to denote $\models(\varphi, w) = 1$):

- $w \models (i)A$	iff	$w(A) = i$
- $w \models \neg P$	iff	$w \not\models P$
- $w \models P \wedge Q$	iff	$w \models P$ and $w \models Q$
- $w \models P \vee Q$	iff	$w \models P$ or $w \models Q$
- $w \models P \rightarrow Q$	iff	$w \not\models P$ or $w \models Q$
- $w \models \Box P$	iff	$w' \models P$, for all $w' \in W$
- $w \models \Diamond P$	iff	$w' \models P$, for some $w' \in W$

where A is an atomic symbol and P and Q are arbitrary *MVEL*-formulas.

Notice that quasi-atoms are not atoms in the classical sense, since they cannot be valued independently. Indeed, they depend on a previous valuation of their propositional variables. However, once quasi-atoms are interpreted, the rest of formulas are interpreted according to the classical two-valued semantics. So, in some sense we have classical interpretations on top of many-valued semantics.

As usual, we can define the standard notions of satisfaction relation and of logical consequence

Definition 2.1 Given a model $K = \langle W, \models, R \rangle$ and a *MVEL*-formula φ, φ is satisfied in K, written $K \models \varphi$, if $w \models \varphi$ for all $w \in W$. Analogously, we write $MVEL \models \varphi$, if $K \models \varphi$ for all *MVEL* models K. Finally, φ is a logical consequence of a set of *MVEL*-formulas Γ, written $\Gamma \models \varphi$, if for all *MVEL* model K, $K \models \psi$ for all $\psi \in \Gamma$ implies $K \models \varphi$.

The particular semantics we have described for *MVEL* leads us to propose for obvious reasons the following axiomatic system, the same as for the classical propositional S5 modal logic, except for the explicit mention to the characteristic relationships between quasi-atoms.

Axioms:
(a) classical tautologies ,
(1) $\bigvee_{i \in A_n} (i)B$, being $B \in At$
(2) $\bigwedge_{i \neq j} \neg((i)B \wedge (j)B)$, being $B \in At$ and $i,j \in A_n$
(b) classical modal axioms for S5

Axioms (1) and (2) state that every propositional variable is given a truth-value and only one.

Deduction rules: modus ponens and necessitation

Definition 2.2 $MVEL \vdash \varphi$ if, and only if, φ follows from the set of axioms of propositional (two-valued) calculus (a), many-valued axioms (1) and (2), modal axioms (b), and by applying the Modus Ponens and Necessitation deduction rules .

Theorem 2.3 (Completeness) $MVEL \vdash \varphi$ iff φ has the value 1 in all worlds of all *MVEL* many-valued Kripke models, i.e. iff $MVEL \models \varphi$

3. *MVEL* Epistemic States

We now want to apply the approach of Halpern and Moses onto the epistemic language *MVEL*. To do so we treat formulas of the form $(z)A$ as 2-valued propositional atoms. In this way we need not really extend the original approach to many-valued logic, since formulas of the above form are 2-valued: they are always true or false. Of course, we should keep in mind that these atoms are not completely independent, but are sometimes logically related, such as given by the axioms (1) and (2). As in the original approach we also consider stable sets of *MVEL*-formulas.

Definition 3.1 A set Φ of *MVEL*-formulas is *stable* if it satisfies the following:

1) closed under *MVEL*-propositional tautologies, that is, Φ includes all classical two-valued propositional tautologies together with the axioms (1) and (2)
2) if $A \in \Phi$ and $A \rightarrow B \in \Phi$ then $B \in \Phi$
3) $A \in \Phi$ iff $\Box A \in \Phi$
4) $A \notin \Phi$ iff $\neg \Box A \in \Phi$
5) Φ is many-valued propositional consistent

Proposition 3.2 Let Φ be *MVEL*-stable. Then there is an *MVEL* Kripke-model K_Φ such that its theory (the formulas true in it) is Φ. In particular $K_\Phi \models$ axiom(1), axiom(2).

Proposition 3.3 An *MVEL*-stable set Φ is closed under S5-consequence.

Stable sets enjoy the following property that we will employ to define minimal epistemic states.

Proposition 3.4 A stable set of epistemic formulas is uniquely determined by the objective formulas it contains.

Now suppose that φ is a (objective) formula that describes all the facts that have been learnt by the agent. We are interested in defining the epistemic state of the agent if he "only knows φ". This state must be minimal in some sense. Halpern & Moses take this epistemic state to be that stable set containing φ for which the objective part is the least (with respect to set inclusion). However such a least stable set does not exist for every formula φ. A formula is called honest if this least stable set does exist. Formally, this is stated by the following definition.

Definition 3.5 Let Prop(Φ) denote the subset of Φ that exactly contains all objective formulas of Φ. A formula φ is *honest* if there exists a stable set Σ^φ that contains φ and such that for all stable sets Σ containing φ it holds that Prop(Σ^φ) $\subseteq$ Prop(Σ).

The intention is that Σ^φ denotes the stable set representing the state of knowledge of the agent who "knows only φ" (if φ is honest). Every objective formula can be proved to be honest, so that we can indeed speak about the epistemic state associated with knowing only some objective formula.

Proposition 3.6 Every objective formula is honest.

The notion of an epistemic state determined by an honest formula naturally leads to a non-monotonic entailment relation which, given an honest formula φ, yields all things that are known in the epistemic state associated with φ.

Definition 3.7 *(non-monotonic entailment)* For an honest φ, we define $\varphi \vdash_{MVEL} \psi$ if, and only if, $\psi \in \Sigma^\varphi$.

4. Linking *MILORD II* to *MVEL*

A Knowledge Base (KB) in *MILORD II* (see Fig. 1) consists of a set of hierarchically interconnected modules. A module can be understood as a functional abstraction, by fixing both the set of components it needs as input and the type of results it can produce.

Each module contains an Object Level Theory (*OLT*) and a Meta-Level Theory (*MLT*) interacting through a reflective mechanism. This meta-level approach, based on reflection techniques and equipped with a declarative backtracking mechanism, is used by the system to deal with non-monotonicity

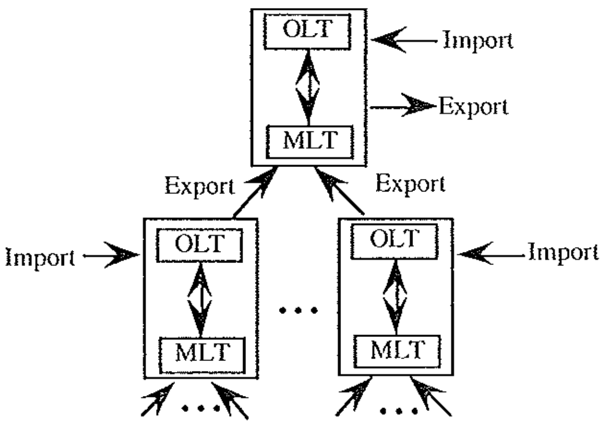

Figure 1. Milord II KB structure.

Reflection mechanisms may be understood, in this context, as a clear separation between domain and control knowledge. Besides, the system provides at the object level a family of representation languages based on multiple-valued logics to deal with approximate reasoning, where the sets of truth-values stand for different scales of linguistic terms representing degrees of truth. Because of the scope of the paper, we will focus on the logical description of different reasoning components of a *MILORD II* module, i.e. the object- level, meta-level and reflection components, as well as on the reasoning dynamics of module as a whole.

The propositional language $OL_n = (A_n, \Sigma_O, C, OS_n)$ of the object level[1] is defined by:

[1]For the sake of simplicity, the syntax used in this Object Level Language description is a simplification of the actual MILORD II syntax.

• A **Signature** Σ_O, composed of a set of atomic symbols plus *true* and *false*.
• A set of **Connectives** $C = \{\neg, \wedge, \rightarrow\}$
• A set of **Sentences** OS_n = *Mv-Atoms* $\cup$ *Mv-Rules*

Sentences are pairs of classical-like propositional sentences and intervals of truth-values. The classical-like propositional sentences are built from a set of atomic symbols and the above set of connectives, but restricted to literals and rules. Thus, the sentences of the language are of the following types:

Mv-Atoms: $\{ (p,V) \mid p \in \Sigma_O$ and V is an interval of truth-values of $A_n\}$
Mv-Literals: $\{ (p,V) \mid (p, V) \in$ *Mv-atoms* or $p = \neg q$ and $(q, V) \in$ *Mv-atoms*$\}$
Mv-Rules: $\{ (p_1 \wedge p_2 \wedge ... \wedge p_n \rightarrow q, V^*)$[2] $\mid p_1, p_2, ..., p_n$ and q are literals, $V = [a, 1]$ is an upper interval of truth-values of A_n, with $a > 0$, and $p_i \neq p_j, p_i \neq \neg p_j, q \neq p_j, q \neq \neg p_j$ for all i and j $\}$

The semantics is basically determined by the *conjunction* and *implication* connective operators of the truth-value algebra. Such operators enjoy most of the properties of usual connectives in Fuzzy Logic since they are the finite counterpart of the well-known *t-norms* and related operators.

Definition 4.1 A *MILORD II algebra of truth-values* is a finite and ordered algebra $A_{n,T} = \langle A_n, 0, 1, T, I_T, N\rangle$ such that:
1) The ordered set of truth-values A_n is a chain of n elements:

$$0 = a_1 < a_2 < ... < a_n = 1$$

where 0 and 1 are the booleans *False* and *True* respectively.
2) The negation operator N_n is the unary operation defined as $N_n(a_i) = a_{n-i+1}$, the only one that fulfils the following properties:
N1: if $a < b$ then $N_n(a) > N_n(b)$, $\forall a,b \in A_n$
N2: $N_n^2 = \mathrm{Id}$.
3) The conjunction operator T is any binary operation such that the following properties hold $\forall a, b, c \in A_n$:
T1: $T(a,b) = T(b,a)$
T2: $T(a,T(b,c)) = T(T(a,b),c)$
T3: $T(0,a) = 0$
T4: $T(1,a) = a$
T5: if $a \leq b$ then $T(a,c) \leq T(b,c)$ for all c
4) The implication operator I_T is defined by residuation with respect to T, i.e.
$I_T(a,b) = Max \{c \in A_n \mid T(a,c) \leq b\}$

From now on, we will take a generic truth-value algebra $A_{n,T} = \langle A_n, 0, 1, T, I_T, N\rangle$ as the truth-value algebra on top of which formulas will be interpreted. Notice that having truth-values in the sentences enables us to define a classical satisfaction relation in spite of the models being multiple-valued assignments.

Models M_ρ are defined by valuations ρ from Σ to A_n such that $\rho(true) = 1$ and $\rho(false) = 0$, and they extend to other first components of sentences in the following way:

$$\rho(\neg p) = N_n(\rho(p))$$
$$\rho(p_1 \wedge p_2 \wedge ... \wedge p_n \rightarrow q) = I_T(T(\rho(p_1), ..., \rho(p_n)), \rho(q))$$

The **Satisfaction Relation** between models and sentences is defined by:

$$M_\rho \models (p, V) \text{ iff } \rho(p) \in V$$

The **Semantical entailment** between sets of sentences Γ and sentences A is defined as usual: $\Gamma \models A$ iff for any model M_ρ, $M_\rho \models \Gamma$ implies $M_\rho \models A$

The object level deduction system $(OL_n, \vdash_O)$ is based on the following axiom scheme:
(AS) $(\varphi, [0, 1])$[3]
on the following axioms
(A-1) $(true, 1)$
(A-2) $(false, 0)$
and on the following inference rules:
(RI-1) **weakening**:
$(p,V_1) \vdash_O (p,V_2)$, where $V_1 \subseteq V_2$, for any literal p
(RI-2) **not-introduction**:
$(p,V) \vdash_O (\neg p, N^*_n(V))$, for any atom p
$(\neg p,V) \vdash_O (p, N^*_n(V))$, for any atom p
(RI-3) **composition**:
$\{(p,V_1), (p,V_2)\} \vdash_O (p,V_1 \cap V_2)$, for any literal p
(RI-4) **specialization**:
$\{(p_i,[a_i, b_i]), (p_1 \wedge p_2 \wedge ... \wedge p_n \rightarrow q,[a_r, 1])\} \vdash_O (p_1 \wedge p_2 \wedge ... \wedge p_{i-1} \wedge p_{i+1} \wedge ... \wedge p_n \rightarrow q,[T(a_i,a_r), 1])$ for any literals p_1 ... p_n and q.

It is easy to check that this deductive system is sound. It is also complete considering some restrictions in the structure of theories [Puyol-Gruart et al, 92].

The meta-level language is based on a first order logic with special metapredicates *WK* and *ASS*. In this paper we will concentrate on the formalization of meta-predicate *WK*. Given a current Object Level Theory (*OLT*), *WK(p,V)* will be true in the corresponding meta-level theory *MLT* when *(p, V)* belongs to *OLT*, i.e. $OLT \vdash_O (p, V)$.

[2] The corresponding rule in MILORD II syntax would be:
If p_1 and p_2 and ... and p_n then conclude q is V*

[3] In any case everyproposition has a truth-value between 0 and 1.

The connexion between object and meta languages is done trough the next reification and reflection rules:

$$\frac{(p,\ V) \in OLT}{\vdash_M WK(p,V)}$$

$$\frac{(p,\ V) \notin OLT}{\vdash_M \neg WK(p,V)}$$

$$\frac{\vdash_M ASS(p,\ V)}{\vdash_O (p,\ V)}$$

Besides meta-predicates *WK* and *ASS*, there also exists meta-predicate *K*. The semantics of $K(p, V)$ is that *V* is the minimal interval such that the proposition (p, V) belongs to the *OLT*. This meta-predicate *K* is definable from meta-predicate *WK* as follows:

$$K(p, [a_i, a_j]) \equiv WK(p, [a_i, a_j]) \wedge$$
$$\wedge \neg WK(p, [a_{i+1}, a_j]) \wedge \neg WK(p, [a, a_{j-1}])$$

MILORD II object-level formulas will be interpreted as objective *MVEL* formulas in a first step. The second step will be to interpret the Meta-level Theory in a particular reasoning stage as the epistemic state determined by the formulas of the current Object-level Theory formulas. We will use the following abbreviations:

- $(\geq a_i)A$ for $(a_i)A \vee (a_{i+1})A \vee ... \vee (1)A = \bigvee_{j \geq i} (a_j)A$,
- $(\leq a_i)A$ for $(0)A \vee (a_1)A \vee ... \vee (a_i)A = \bigvee_{j \leq i} (a_j)A$
- $(a_i{:}a_j)A$ for $(a_i)A \vee (a_{i+1})A \vee ... \vee (a_j)A = \bigvee_{i \leq k \leq j} (a_k)A$

and specially, the following abbreviations are important to understand the intuition behind the embedding of *MILORD II* into *MVEL*:

- $(z)(\neg A)$ for $(N(z))A$
- $(z)(A \wedge B)$ for $\bigvee_{T(x,y)=z} (x)A \wedge (y)B$
- $(z)(A \vee B)$ for $\bigvee_{S(x,y)=z} (x)A \wedge (y)B$
- $(z)(A \rightarrow B)$ for $\bigvee_{I(x,y)=z} (x)A \wedge (y)B$

where $S(x,y) = N(T(N(x),N(y)))$. Notice that introducing these abbreviations into the fragment of non-modal *MVEL* formulas amounts to consider a propositional language $L(At)$ built in the usual way from the set of propositional symbols *At*, unary connectives such as $\neg$, interval indicators (V), being *V* any interval of truth-values, and from the binary connectives $\wedge$, $\vee$, and $\rightarrow$.

Due to the many-valued nature of both MILORD and *MVEL*, it is easy to establish a one-to-one correspondence between *MVEL* possible worlds and *MILORD II* object-level models when the corresponding languages are built over the same set of atomic symbols, denoted Σ_O in *MILORD II* and *At* in *MVEL* (see section 2).

The above abbreviations give the hint of how to translate *MILORD II* object-level formulas into *MVEL* formulas. We will denote by φ^* the translation of a *MILORD II* formula φ following the next table schema.

MILORD II $\Rightarrow$	*MVEL*
mv-literals: $(p, [a_i, a_j])$	$(a_i{:}a_j)p$
mv-rules: $(p_1 \wedge ... \wedge p_n \rightarrow q, [a_i, 1])$	$(\geq a_i)(p_1 \wedge ... \wedge p_n \rightarrow q)$

In order to make explicit the semantical equivalence of the pairs of formulas in the above table, let $\Sigma_O = At$, and denote by w both a *MVEL* possible world and the corresponding *MILORD II* model. We will write $\models_{MILORD}$ and $\models_{MVEL}$ to differentiate the MILORD and *MVEL* entailment relations respectively. First, it is easy to see that the next lemma holds.

Lemma 4.2 For any $A, B \in L(At)$ and for any possible world w, the following properties hold:
- $w \models_{MVEL} (z)(\neg A)$ iff $z = N(w(A))$
- $w \models_{MVEL} (z)(A \wedge B$ iff $z = T(w(A) \wedge w(B))$
- $w \models_{MVEL} (z)(A \vee B)$ iff $z = S(w(A) \wedge w(B))$
- $w \models_{MVEL} (z)(A \rightarrow B)$ iff $z = I(w(A) \wedge w(B))$

where w is supposed to be extended to non-atomic sentences of $L(At)$ using opertaions *N*, *T*, *S* and *I* to interpret negation, conjunction, disjunction and implication respectively.

As a consequence of this lemma, next proposition proves that the embedding of the *MILORD II* object language in *MVEL* is faithful.

Proposition 4.3 For all $\varphi \in$ *Mv-literals* $\cup$ *Mv-rules*, we have: $w \models_{MILORD} \varphi$ if, and only if, $w \models_{MVEL} \varphi^*$, where φ^* is the translation of φ according to the above table.

As a consequence of this theorem, we can consider the *MILORD II* object-level Language as a fragment of the *MVEL* language consisting of formulas of type $(\geq z)A$, or more generally, of formulas of type $(x{:}y)A$, where *A* is an objective implication with a conjunction (possibly empty) of literals as premise and with a literal as conclusion. Next section is devoted to extend the embedding when the *MILORD II* meta-level language is also considered.

5. Modal Interpretation of K and WK Meta-Predicates

Our aim in this paper is to prove that meta-predicates can be given a modal interpretation when the meta-level architecture of *MILORD II* is used to model non-monotonic epistemic reasoning in the sense that the meta-level component system is to reason about the state of knowledge of the object-level component. A MILORD-II reasoning flow inside a module can be modelled as a sequence of object and meta level theories as depicted in Fig. 2.

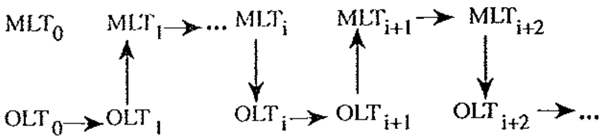

Figure 2. Dynamics of the reasoning process in MILORD II. The symbol → stands for deductive closure (w.r.t the corresponding languages), ↑ for the upwards reflection and ↓ for the downwards reflection.

In this setting, the object-level theorie at stage i OLT_i is considered to be the closure, w.r.t. the object-level deduction system (see section 4), of OLT_{i-1}. On the other hand, it is also quite natural to represent the state of knowledge of the object-level component at a given step i by the set of mv-literals OLT_i contains. This would be very close to the many-valued counterpart of the concept of partial model used in the *BMS* system (Tan, 92). More concretely, if we denote by $Lit(OLT_i)$ the set of mv-literals belonging to OLT_i, the corresponding meta-theory MLT_i is built as follows:

$$MLT_i = MLT_o \cup \{ WK(A) \mid A \in Lit(OLT_i) \} \cup \{\neg WK(A) \mid A \notin Lit(OLT_i) \}$$

where MLT_o is the set of initial meta-rules of the Knowledge Base. Notice that, because of being OLT_i deductively closed, and in particular under the *weakening* rule of inference RI-1, if $(p, V) \in Lit(OLT_i)$, then $(p, U) \in Lit(OLT_i)$ for all $U \supseteq V$. The idea to modally interpret the meta-predicate WK (the K predicate can be defined in terms of WK) is to consider the proposition

$$\varphi_i = \bigwedge \{ A^* \mid A \in Lit(OLT_i) \}$$

describing the state of knowledge of the object level system at the step i, where A^* denotes the translation of A to its corresponding $MVEL$ formula (see previous section). Inside $MVEL$, φ_i is a finite conjunction of objective formulas and thus it is honest. Therefore, it makes sense to consider the epistemic state Σ^{φ_i}. In this context, the modal interpretation of meta-predicate WK is given by the following theorem.

Theorem 5.1. For any literal p it holds:
$WK(p, [a_i, a_j]) \in MLT_i$ iff $\Box(a_i{:}a_j)p \in \Sigma^{\varphi_i}$
or equivalently, in terms of the non-monotonic entailment:
$WK(p, [a_i, a_j]) \in MLT_i$ iff $\varphi_i \mid\!\sim_{MVEL} \Box(a_i{:}a_j)p$

Recall that *MILORD II* upward reflection rule (see section 4) provides the semantical equivalence between the formulas $K(p, [a_i, a_j])$ and $WK(p, [a_i, a_j]) \wedge \neg WK(p, [a_{i+1}, a_j]) \wedge \neg WK(p, [a, a_{j-1}])$. As a direct consequence, the next corollary provides the modal interpretation of the *MILORD II* meta-predicate K.

Corollary 5.2. For any literal p it holds:
$K(p, [a_i, a_j]) \in MLT_i$ iff
$\Box(a_i{:}a_j)p \wedge \neg\Box(a_{i+1}{:}a_j)p \wedge \neg\Box(a_i{:}a_{j-1})p \in \Sigma^{\varphi_i}$

The preliminary results of the above theorem and corollary show that it actually makes sense to provide a modal interpretation for the meta-predicates, shown in next table that completes the one given in the previous section. However, further research is needed.

MILORD II ⇒	*MVEL*
mv-literals: $(p, [a_i, a_j])$	$(a_i{:}a_j)p$
mv-rules: $(p_1 \wedge ... \wedge p_n \to q, [a_i, 1])$	$(\geq a_i)(p_1 \wedge ... \wedge p_n \to q)$
mv-meta-literals: $WK(p, [a_i, a_j])$, $K(p, [a_i, a_j])$	$\Box(a_i{:}a_j)p$, $\Box(a_i{:}a_j)p \wedge \neg\Box(a_{i+1}{:}a_j)p \wedge \neg\Box(a_i{:}a_{j-1})p$
mv-meta-rules[4]: $WK(p,[a,1]) \wedge \neg WK(q,[b,1]) \to ASS(r, [c,1])$	$\Box(\geq a)p \wedge \neg\Box(\geq b)q \to \Delta(\geq c)r$

As a summary, we can consider the *MILORD II* Object and Meta Level Languages as fragments of *MVEL*, namey:

- The Object Level Language corresponds to formulas of type $(\geq z)A$, or more generally $(x{:}y)A$, where A is an objective implication with a conjunction (possibly empty) of literals as.premise and with a literal as conclusion.
- The Meta Level Language corresponds to epistemic implicative formulas of the general form $\Box(\geq x)p \wedge \neg\Box(\geq y)q \to \Delta(\geq z)r$.

6. Example

Consider the next example of a *MILORD II* KB.

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[4] mv-meta-rules, the behavior of ASS and its corresponding Δ modal operator and their role in the reflection down process are not dealt within this paper (see [Meyer & van der Hoek, 93b], for a possible treatment of a reflection down process).

Truth-values {0 = false < moderately-possible (mp) < possible (p) < very-possible (vp) < certain = 1}

Connectives: T(x,y) = min(x,y)

I(x,y) = 1, if $x \leq y$; I(x,y) = y, otherwise

Object level formulas (mv-literals, mv-rules):

(Low_WBC, [vp, 1])

(Low_WBC → Compromised_Host, [vp, 1)

(Gram_neg_inf → Est_Coli, [p, 1])

(I-Dont_know → Gram_Pos_inf, [vp, 1])

Meta Level formulas (mv-meta-rules):

WK(Compromised_Host, [p, 1]) ∧

¬ WK(Gram_Pos_inf, [modp, 1]) →

ASS(Gram_neg_inf, [vp, 1])

This KB is rewritten in *MVEL* formalism as

Object level (Objective formulas):

(≥vp) low_WBC

(≥vp) (low_WBC → Compromised_Host)

(≥p) (Gram_neg_inf → Est_Coli)

(≥vp) (I-Dont_know → Gram_Pos_inf)

Meta level (Modal Formulas):

□ (≥p) Compromised_host ∧

¬ □ (≥modp) Gram_Pos_inf → Δ (≥vp) Gram_Neg_inf

The dynamics of the reasoning process is represented in the next figure,

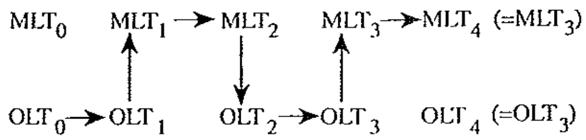

Figure 3. Dynamics of the reasoning process in the example considered.

and described below.

Begin

1. $OLT_0 = KB_{object}$, $MLT_0 = KB_{meta}$
2. $OLT_1 = OLT_0$ + {(≥vp)Compromised_Host}
3. $MLT_1 = MLT_0$ + {□ (≥vp)Compromised_host, □(≥vp)low_WBC} + {¬□ $(a_i{:}a_j)$Gram_Pos_inf, ¬□$(a_i{:}a_j)$Gram_neg_inf, ¬□$(a_i{:}a_j)$Est_Coli, ¬□ $(a_i{:}a_j)$I-Dont_know | $(a_i{:}a_j) \neq (0{:}1)$}
4. $MLT_2 = MLT_1$ + {Δ (≥vp) Gram_Neg_inf}
5. $OLT_2 = OLT_1$ + {(≥vp) Gram_Neg_inf}
6. $OLT_3 = OLT_2$ + {(≥p) Est_Coli}
7. $MLT_3 = MLT_0$ + {□ (≥vp)Compromised_host, □(≥vp)low_WBC} + {□ (≥vp) Gram_Neg_inf, □ (≥p) Est_Coli} + {¬□ $(a_i{:}a_j)$Gram_Pos_inf, ¬□ $(a_i{:}a_j)$I-Dont_know | $(a_i{:}a_j) \neq (0{:}1)$}
8. $MLT_4 = MLT_3$
9. $OLT_4 = OLT_3$

Stop

In terms of epistemic states, this reasoning flow would be equivalently expressed as follows:

a. $KB_{object} \vdash$ (≥vp)Compromised_host

b. φ_1 = (≥vp) low_WBC (≥vp) ∧ (≥vp)Compromised_host

c. $\varphi_1 \vdash$□ (≥vp)Compromised_host (or equivalently, □(≥vp)Compromised_host $\in \Sigma^{\varphi 1}$)

$\varphi_1 \vdash \neg$□ (≥modp) Gram_Pos_inf (or equivalently, ¬□ (≥modp) Gram_Pos_inf $\in \Sigma^{\varphi 1}$)

d. $\varphi_1 \cup KB_{meta} \vdash$Δ (≥vp) Gram_Neg_inf

e. $KB_{object} \cup$ {(≥vp) Gram_Neg_inf} $\vdash$ (≥p) Est_Coli

f. $\varphi_2 = \varphi_1$ ∧ (≥vp) Gram_Neg_inf ∧ (≥p) Est_Coli

g. $\varphi_2 \vdash$□(≥p) Est_Coli

7. Conclusions

In this paper a many-valued extension of the concept of epistemic states is presented. Its application to the meta-level architecture *MILORD II* is developed by giving a modal epistemic interpretation of some meta-predicates that allow reasoning about the object level knowledge state. This work has a clear relation with the formalization of default reasoning via epistemic logics as in [Meyer & van der Hoek, 93a,93b]

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