Extending a Temporal Defeasible Argumentation Framework with Possibilistic Weights

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Abstract. Recently, a temporal extension of the argumentation defeasible reasoning system DeLP has been proposed. This system, called t-DeLP, allows to reason defeasibly about changes and persistence over time but does not offer the possibility of ranking defeasible rules according to criteria of preference or certainty (in the sense of belief). In this contribution we extend t-DeLP by allowing to attach uncertainty weights to defeasible temporal rules and hence stratifying the set of defeasible rules in a program. Technically speaking, weights are modelled as necessity degrees within the frame of possibility theory, a qualitative model of uncertainty.

1 Introduction and Motivation

The system DeLP [13] provides a defeasible logic programming-based argumentation framework, based on the use of dialectical trees, upon which several extensions have been built. In particular, the system t-DeLP [17] makes it possible to express defeasible reasoning in a discrete temporal framework as well as changes and persistence over time. t-DeLP, however, does not offer the possibility of ranking defeasible rules according to criteria of preference or certainty (in the sense of belief).

In addition, in t-DeLP, the application of rules does not take into account any possible uncertainty in the occurrence of temporal events. This issue has been already tackled in another extension of DeLP, the system P-DeLP [3,2,8], that allows the handling of possibilistic uncertainty (of a qualitative, ordinal nature) by attaching defeasible rules and arguments with necessity degrees.

To bridge this gap, in this paper we introduce an extended argumentation framework, called pt-DeLP, where it is possible to express uncertainty on temporal rules and events, and how this uncertainty may change over time. This new system presents the possibility of formalizing arguments and defeat relations among them that combine both temporal criteria of t-DeLP and the belief strength criteria from P-DeLP. In fact, preferred arguments in t-DeLP are those with comparably more (temporally basic) information, or comparably less use of

L. Fariñas del Cerro, A. Herzig, and J. Mengin (Eds.): JELIA 2012, LNCS 7519, pp. 242–254, 2012.

persistence rules (which simply assume that some fact will not change). This allows to express certain non-monotonic temporal phenomena (extinction of facts vs. persistence) in an economic way. On the other hand, P-DeLP establishes a preference for arguments that support their conclusions with comparably higher weights. Then, a natural way of combining these two kinds of preference (defeat) relations is through the definition of a lexicographic preference relation (see e.g. [4]) that assigns more relevance to temporal criteria or vice versa. We will show that, under any of these two lexicographic combinations, pt-DeLP is a conservative extension of t-DeLP, while this is not the case with P-DeLP.

The paper is structured as follows. In Section 2 we present the logic underlying the proposed logic programming framework, which is introduced in Section 3. The relationship between t-DeLP and P-DeLP to the new framework pt-DeLP is studied in Section 4. Then, an illustrative example is developed in Section 5 and we finish with a brief discussion on related work and conclusions.

2 Language and Semantics of the Base Logic of pt-DeLP

The logic upon which pt-DeLP is based on consists of (Boolean combinations of) temporal formulas, encoding the occurrence of an event at a given time, equipped with a weight that represents the degree of uncertainty attached to the formula. Here, we describe how pt-DeLP is defined within such a syntactic framework.

Given a finite set of propositional variables $\mathsf{Var} = \{p, q, \dots\}$, let us denote by Lit the set of literals built from Var, i.e. $\mathsf{Lit} = \{p, \neg p \mid p \in \mathsf{Var}\}$. By $\neg\ell$ we will denote $\neg p$ if $\ell = p$, or p if $\ell = \neg p$, with $p \in \mathsf{Var}$.

The set ATForm is defined as the set of all pairs (ℓ, t) such that $\ell \in \mathsf{Lit}$ and $t \in \mathbb{N}$. Every formula in ATForm is called an *atomic temporal formula*. The set TForm of *temporal formulas* is built, as usual, from the set of atomic temporal formulas with the classical Boolean connectives $\wedge, \vee, \neg, \longrightarrow$.

A temporal interpretation for TForm is a mapping $w : \mathsf{ATForm} \times \mathbb{N} \to \{0, 1\}$ such that, for each $t \in \mathbb{N}$, $w((\neg p, t)) = 1 - w((p, t))$. The interpretation w extends to formulas TForm as usual using the classical truth-functions for the Boolean connectives. Notice that the above extra condition ensures that, in fact, temporal formulas $(\neg\ell, t)$ and $\neg(\ell, t)$ are considered as logically equivalent.

We will denote by Ω the set of temporal interpretations over TForm. An interpretation $w \in \Omega$ is called a *model* of a temporal formula Φ, denoted by $w \models \Phi$, whenever $w(\Phi) = 1$.

In what follows, we expand the temporal language by introducing weights and provide a suitable semantics in terms of possibilistic uncertainty.[1]

Definition 1. *A* possibilistic model *over* TForm *is a possibility distribution* $\pi : \Omega \to [0, 1]$ *such that* $\max_{w \in \Omega} \pi(w) = 1$. *The possibility distribution* π *induces a necessity measure* N_π *on* TForm *in the usual way, i.e.:* $N_\pi(\Phi) = \inf\{1 - \pi(w) \mid w \in \Omega, w \not\models \Phi\}$.

[1] For all the details on possibility theory and possibilistic logic the reader is referred e.g. to [11] and the references therein.

A *weighted temporal formula* (*wt*-formula) is an expression of the form $\langle \Phi \rangle_r$, with $\Phi \in \mathsf{TForm}$, and $r \in [0,1]$, which is to be interpreted as a lower bound for the necessity degree of Φ. Note that a formula like $\langle (\ell_1, t) \longrightarrow (\ell_2, t) \rangle_r$ is a formula from TForm, while $\langle (\ell_1 \longrightarrow \ell_2, t) \rangle_r$ is not. The set of all *wt*-formulas is denoted WTForm.

Definition 2. *A* possibilistic model π *over* TForm *satisfies a weighted temporal formula* $\langle \Phi \rangle_r$*, denoted* $\pi \models_{pos} \langle \Phi \rangle_r$*, whenever* $N_\pi(\Phi) \geq r$*. As usual, we say that* π *satisfies a set of wt-formulas* P *whenever* π *satisfies every* $\langle \Phi \rangle_r \in P$*. Moreover, the induced consequence relation is defined as follows:* $P \models_{pos} \langle \Phi \rangle_r$ *iff every* π *satisfying* P *also satisfies* $\langle \Phi \rangle_r$*.*

We now define the language of $\mathsf{pt\text{-}DeLP}$ as a fragment of WTForm, replacing $\longrightarrow$ by $\longleftarrow$ as costumary in logic programming languages. Let

$$\mathsf{WTLit} = \{ \langle (\ell, t) \rangle_r \mid \ell \in \mathsf{Lit}, t \in \mathbb{N}, r \in [0,1] \}$$

be the set of all *wt*-formulas $\langle \Phi \rangle_r$ where Φ is in ATForm. Each formula of this kind is called a *weighted temporal literal* [2] (*wt*-literal, for short). Moreover, let

$$\mathsf{WTRule} = \{ \langle (\ell, t) \longleftarrow (\ell_1, t_1) \wedge \cdots \wedge (\ell_n, t_n) \rangle_r \mid (\ell, t), \ldots, (\ell_n, t_n) \in \mathsf{ATForm}; \\ t \geq \max\{t_1, \ldots, t_n\}, r \in [0,1] \}$$

be the set of all *wt*-formulas $\langle \Phi \rangle_r$ where Φ is a temporal formula of the form $(\ell, t) \longleftarrow (\ell_1, t_1) \wedge \cdots \wedge (\ell_n, t_n)$. Each formula contained in WTRule is called a *weighted temporal rule* (*wt*-rule, for short).[3] If $\delta = \langle (\ell, t) \longleftarrow (\ell_1, t_1) \wedge \cdots \wedge (\ell_n, t_n) \rangle_r$, we write $\mathsf{head}(\delta) = (\ell, t)$, $\mathsf{body}(\delta) = \{(\ell_1, t_1), \ldots, (\ell_n, t_n)\}$, and $\mathsf{lit}(\delta) = \{\mathsf{head}(\delta)\} \cup \mathsf{body}(\delta)$. If $r = 1$, δ is called a *strict* rule, and *defeasible* otherwise (i.e. when $0 < r < 1$). The language of $\mathsf{pt\text{-}DeLP}$ corresponds to the fragment of WTForm consisting of the set of formulas $\mathsf{WTLit} \cup \mathsf{WTRule}$.

Note that rules from WTRule are forward temporal rules (i.e rules in which the time of the occurrence of the literal in the conclusion follows the time of the occurrences of the premises), and so we keep from $\mathsf{t\text{-}DeLP}$ the idea that temporal rules represent causal relationships. For instance, a *wt*-rule like $\langle (\mathsf{dead}(\text{Lars}), t) \longleftarrow (\mathsf{bitten}(\text{Lars}), t_1) \wedge (\mathsf{antidote}(\text{Lars}), t_2) \rangle_\alpha$ means: it is α-plausible that Lars dies at t when poisoned at t_1 and given an antidote at t_2.

It turns out that in many situations the weight attached to a temporal formula $(\ell, t) \longleftarrow (\ell_1, t_1) \wedge \cdots \wedge (\ell_n, t_n)$ does not actually depend on the absolute time values but on the temporal distances $t - t_1, \ldots, t - t_n$ between the time of occurrence of the head of the rule and of the different premises. For this reason we introduce the following convenient and schematic notation for specifying a given temporal evolution of the weights in a *wt*-rule. In a sense, such schematic representations assume that the weights are invariant under uniform time translations: a *schematic* weighted temporal rule is an expression δ_ν of the form

$$\langle \ell \longleftarrow (\ell_1, \mathsf{d}_1) \wedge \cdots \wedge (\ell_n, \mathsf{d}_n) \rangle_{\nu(\mathsf{d}_1, \ldots, \mathsf{d}_n)},$$

[2] In the rest of the work, *wt*-literals $\langle (\ell, t) \rangle_r$ will be written in the simpler form $(\ell, t)_r$.

[3] The notation $\langle (\ell, t) \longleftarrow (\ell_1, t_1), \ldots, (\ell_n, t_n) \rangle_r$ will also be used later for *wt*-rules.

where $\mathtt{d}_1, \ldots, \mathtt{d}_n$ are variables taking values in $\mathbb{N}$ and $\nu : \mathbb{N}^n \to [0,1]$. This schematic rule compactly encodes the set of the following instantiated *wt*-rules:

$$\delta = \langle (\ell, t) \longleftarrow (\ell_1, t - d_1) \wedge \cdots \wedge (\ell_n, t - d_n) \rangle_{\nu(d_1, \ldots, d_n)}$$

for each $d_1, \ldots, d_n \in \mathbb{N}$ and for each $t \geq \max\{d_1, \ldots, d_n\}$. The function ν is called a *weight distribution* and assigns to each instantiated rule δ a weight $\nu(d_1, \ldots, d_n)$ that depends on the temporal distances $d_1, \ldots, d_n$ of the head of the rule ℓ with respect to each premise ℓ_i in the body.

As a particular case, we can represent weighted versions of t-DeLP persistence rules δ_ℓ for selected literals ℓ. These are of the form $\langle \ell \longleftarrow (\ell, \mathtt{d}) \rangle_{\nu(\mathtt{d})}$ which state that a literal ℓ holding at t will still hold at $t + d$ with degree $\nu(d)$, for any d.

The notion of derivability in pt-DeLP is the natural extension of that of t-DeLP with possibilistic weights by means of a weighted version of modus ponens (which is sound with respect to the above possibilistic semantics, see e.g. [11]).

Definition 3 (Derivability). *Given a set of wt-literals and wt-rules* $P \subseteq$ WTLit $\cup$ WTRule*, we say that a wt-literal* $(\ell, t)_r$ *is* derivable *from* P*, denoted* $(\ell, t)_r \in Cn(P)$*, iff*

1. $(\ell, t)_{r'} \in P$ *with* $r \leq r'$*, or*
2. *there exist a set of wt-literals* $(\ell_1, t_1)_{r_1}, \ldots, (\ell_n, t_n)_{r_n} \in Cn(P)$ *and a wt-rule*

$$\langle (\ell, t) \longleftarrow (\ell_1, t_1) \wedge \cdots \wedge (\ell_n, t_n) \rangle_s, \qquad \text{such that } r \leq \min\{s, r_1, \ldots, r_n\}.$$

3 An Argumentation System for pt-DeLP

Logic-based argumentation systems aim at providing computational tools to reason under conflicting or inconsistent information. Therefore, it is crucial to have a clear notion of what an inconsistent set of pt-DeLP formulas is.

Definition 4. *A set* $P \subseteq$ WTLit $\cup$ WTRule *is said to be pt-DeLP-inconsistent if there exist* $(\ell, t)_r, (\neg\ell, t)_s \in Cn(P)$ *with* $\min(r, s) > 0$*. Otherwise, we say that* P *is pt-DeLP-consistent.*

Note that if P is inconsistent, P is not satisfiable by any possibilistic model since $\min(N_\pi((\ell, t)), N_\pi((\neg\ell, t))) = 0$ in any possibilistic model π. The converse is not true, since the notion of derivability in pt-DeLP is obviously weaker than the possibilistic logical consequence $\models_{pos}$. In other words, if $(\ell, t)_r \in Cn(P)$ then $P \models_{pos} (\ell, t)_r$, but the opposite does not always hold.

Definition 5 (Program). *A* pt-DeLP program *is a pair* (Π, Δ) *such that* Π *is a consistent finite set of strict wt-rules and* Δ *is a finite set defeasible wt-rules.*

Definition 6 (Argument). *Given a pt-DeLP program* $P = (\Pi, \Delta)$*, an* argument *for* $(\ell, t)_r$ *is a set* $\mathcal{A} = \mathcal{A}_\Pi \cup \mathcal{A}_\Delta$*, with* $\mathcal{A}_\Pi \subseteq \Pi$ *and* $\mathcal{A}_\Delta \subseteq \Delta$*, such that:*

1. $\Pi \cup \mathcal{A}_\Delta$ *is pt-DeLP-consistent,*

2. $r = \max\{s \in [0,1] \mid (\ell,t)_s \in Cn(\mathcal{A}_\Delta \cup \mathcal{A}_\Pi)\}$,
3. *Both* $\mathcal{A}_\Delta$ *and* $\mathcal{A}_\Pi$ *are minimal w.r.t. inclusion, i.e.: there are no* $\mathcal{A}'_\Delta \subset \mathcal{A}_\Delta$ *and* $\mathcal{A}'_\Pi \subset \mathcal{A}_\Pi$ *such that* $(\ell,t)_r \in Cn(\mathcal{A}'_\Delta \cup \mathcal{A}'_\Pi)$.

Notice that, given a pt-DeLP program (Π,Δ) there may exist different arguments for $(\ell,t)_r$, with different r. In fact, there might be different sets $\mathcal{A} = \mathcal{A}_\Pi \cup \mathcal{A}_\Delta$ and $\mathcal{A}' = \mathcal{A}'_\Pi \cup \mathcal{A}'_\Delta$, with $\mathcal{A}_\Pi \not\subseteq \mathcal{A}'_\Pi$ and $\mathcal{A}_\Delta \not\subseteq \mathcal{A}'_\Delta$, such that $\mathcal{A}$ is an argument for $(\ell,t)_s$, while $\mathcal{A}'$ is an argument for $(\ell,t)_{s'}$, with $s \neq s'$.

Definition 7. *Given a pt-DeLP program* (Π,Δ) *and an argument* $\mathcal{A} = \mathcal{A}_\Pi \cup \mathcal{A}_\Delta$ *for* $(\ell,t)_r$, *we define:*

1. $\mathsf{concl}(\mathcal{A}) = (\ell,t)_r$;
2. $\mathsf{base}(\mathcal{A}) = \{(\ell',t) \mid \exists\delta \in \mathcal{A}, (\ell',t) \in \mathsf{body}(\delta),\ \textit{and}\ \ \nexists\delta \in \mathcal{A}, (\ell',t) \in \mathsf{head}(\delta)\}$;
3. $\mathsf{TLit}(\mathcal{A}) = \bigcup_{\delta\in\mathcal{A}} \mathsf{lit}(\delta)$.

Definition 8 (Sub-argument). *Let* (Π,Δ) *be a pt-DeLP program and let* $\mathcal{A} = \mathcal{A}_\Pi \cup \mathcal{A}_\Delta$ *be an argument for* $(\ell,t)_r$ *in* (Π,Δ). *Given some* $(\ell_0,t_0) \in \mathsf{TLit}(\mathcal{A})$, *a* sub-argument *for* $(\ell_0,t_0)_s$ *(for some* $s \in (0,1]$*) is a set* $\mathcal{B} = \mathcal{B}_\Pi \cup \mathcal{B}_\Delta$, *with* $\mathcal{B}_\Delta \subseteq \mathcal{A}_\Delta$ *and* $\mathcal{B}_\Pi \subseteq \mathcal{A}_\Pi$, *such that* $\mathcal{B}$ *is an argument for* $(\ell_0,t_0)_s$. *The sub-argument of* $\mathcal{A}$ *induced by* (ℓ_0,t_0) *will be denoted* $\mathcal{A}(\ell_0,t_0)$.

We now define the notion of attack between arguments as a natural extension of those for DeLP and t-DeLP.

Definition 9 (Attack). *Given a pt-DeLP program* (Π,Δ), *let* $\mathcal{A}_0$ *and* $\mathcal{A}_1$ *be arguments for* $(\ell_0,t_0)_{r_0}$ *and* $(\ell_1,t_1)_{r_1}$, *respectively. We say that* $\mathcal{A}_1$ attacks $\mathcal{A}_0$ *iff there exists a subargument* $\mathcal{B}_0$ *of* $\mathcal{A}_0$ *for a weighted temporal literal* $(\neg\ell_1,t_1)_s$ *for some* $s > 0$. *In this case,* $\mathcal{A}_1$ *is said to attack* $\mathcal{A}_0$ *at* $\mathcal{B}_0$.

Given the notion of attack, we can now consider several notions of defeat. On the one hand, following the idea proposed in P-DeLP (Possibilsitic DeLP), the presence of weights makes it reasonable to use such weights in deciding whether an attacking argument defeats another argument. Note that the next definition is a direct extension to pt-DeLP of the notion of defeater in P-DeLP.

Definition 10 (Possibilistic Defeater). *Let an argument* $\mathcal{A}_1$ *attack* $\mathcal{A}_0$ *at* $\mathcal{B}_0$, *where we have* $\mathsf{concl}(\mathcal{A}_1) = (\ell,t)_r$ *and* $\mathsf{concl}(\mathcal{B}_0) = (\neg\ell,t)_s$. *We say that* $\mathcal{A}_1$ *is a* proper possibilistic defeater *for* $\mathcal{A}_0$, *denoted* $\mathcal{A}_1 \succ_\pi \mathcal{A}_0$, *whenever* $r > s$; *and we say that* $\mathcal{A}_1$ *is a* blocking possibilistic defeater *for* $\mathcal{A}_0$, *denoted* $\mathcal{A}_1 \preceq\succeq_\pi \mathcal{A}_0$, *whenever* $s = r$.

Still, time and information specificity plays a fundamental role in reasoning in pt-DeLP, and so the definition given in [17] of a *temporal* defeater for t-DeLP should be also taken into account. In the following definition $\Delta_{\neg\ell}$ denotes a suitable set of instances of the schematic persistence rule $\langle\neg\ell \longleftarrow (\neg\ell,\mathsf{d})\rangle_{\nu(\mathsf{d})}$.

Definition 11 (Temporal Defeater). *Let an argument* $\mathcal{A}_1$ *attack* $\mathcal{A}_0$ *at* $\mathcal{B}_0$, *where* $\mathsf{concl}(\mathcal{A}_1) = (\ell,t)_r$ *and* $\mathsf{concl}(\mathcal{B}_0) = (\neg\ell,t)_s$. *We say that* $\mathcal{A}_1$ *is a* proper temporal defeater *for* $\mathcal{A}_0$, *denoted* $\mathcal{A}_1 \succ_\tau \mathcal{A}_0$ *iff*

1. $\mathsf{base}(\mathcal{A}_1) \supsetneq \mathsf{base}(\mathcal{B}_0)$, *or*
2. *there exists* $t' < t$ *such that* $\mathcal{B}_0 = \mathcal{A}_1(\ell, t') \cup \Delta_{\neg\ell}$

We say $\mathcal{A}_1$ *is a* blocking temporal defeater *for* $\mathcal{A}_0$*, denoted* $\mathcal{A}_1 \preceq\succeq_\tau \mathcal{A}_0$ *iff neither* $\mathcal{A}_1$ *nor* $\mathcal{B}_0$ *are proper temporal defeaters for each other.*

The fact that pt-DeLP is built upon the presence of time and weights, formalized in terms of a necessity measures, suggests that both the concepts of possibilistic and a temporal defeaters should be taken into account to define a proper notion of defeat.

Definition 12 (($\pi \times \tau$)-defeater, ($\tau \times \pi$)-defeater). *Let an argument* $\mathcal{A}_1$ *attack* $\mathcal{A}_0$ *at* $\mathcal{B}_0$*, where* $\mathsf{concl}(\mathcal{A}_1) = (\ell, t)_r$ *and* $\mathsf{concl}(\mathcal{B}_0) = (\neg\ell, t)_s$*. We say that* $\mathcal{A}_1$ *is a* proper $(\pi \times \tau)$-defeater *for* $\mathcal{A}_0$*, denoted* $\mathcal{A}_1 \succ_{\pi\times\tau} \mathcal{A}_0$ *iff*

1. $\mathcal{A}_1 \succ_\pi \mathcal{A}_0$*, i.e.* $\mathcal{A}_1$ *is a* proper possibilistic defeater*, or*
2. $\mathcal{A}_1 \preceq\succeq_\pi \mathcal{A}_0$*, i.e.* $\mathcal{A}_1$ *is a* blocking possibilistic defeater *for* $\mathcal{A}_0$*, and* $\mathcal{A}_1 \succ_\tau \mathcal{A}_0$*, i.e.* $\mathcal{A}_1$ *is a* proper temporal defeater *for* $\mathcal{A}_0$*.*

We say that $\mathcal{A}_1$ *is a* blocking $(\pi \times \tau)$-defeater *for* $\mathcal{A}_0$*, denoted* $\mathcal{A}_1 \preceq\succeq_{\pi\times\tau} \mathcal{A}_0$ *iff* $\mathcal{A}_1 \preceq\succeq_\pi \mathcal{A}_0$ *and* $\mathcal{A}_1 \preceq\succeq_\tau \mathcal{A}_0$*. The definition of proper and blocking* $(\tau \times \pi)$*-defeater is completely analogous and is obtained by replacing* τ *with* π *and vice versa.*

The argumentation semantics for pt-DeLP inherits the one of DeLP (as well as those of t-DeLP and P-DeLP) based on dialectical trees, with slight modifications in each case. The following definitions actually are parametric with respect to the notion of proper and blocking defeater, so they can be instantiated in any of the $(\tau \times \pi)$ or $(\pi \times \tau)$ criteria. Of course, depending on whether we prioritize the temporal defeat criteria or the comparison among weights, we will obtain distinct notions of warrant.

Definition 13 (Argumentation Line, Dialectical Tree, Marking). *Let* $\mathcal{A}_1$ *be an argument in* (Π, Δ)*. An* argumentation line *for* $\mathcal{A}_1$ *is a sequence* $\Lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots]$ *where:*

- (i) *supporting arguments, i.e. those in odd positions* $\mathcal{A}_{2i+1} \in \Lambda$ *are jointly consistent with* Π*, and similarly for interfering arguments* $\mathcal{A}_{2i} \in \Lambda$ *;*
- (ii) *a supporting (resp. interfering) argument is different from the attacked sub-arguments of previous supporting (resp. interfering) arguments:* $\mathcal{A}_{i+2k} \neq \mathcal{A}_i(\neg\mathsf{concl}(\mathcal{A}_{i+1}))$*;*
- (iii) $\mathcal{A}_{i+1}$ *is a proper defeater for* $\mathcal{A}_i$ *if* $\mathcal{A}_i$ *is a blocking defeater for* $\mathcal{A}_{i-1}$*.*

An argumentation line $[\mathcal{A}_1, \ldots, \mathcal{A}_n]$ *for* $\mathcal{A}_1$ *is* maximal *if there is no other argument* $\mathcal{A}_{n+1}$ *such that* $[\mathcal{A}_1, \ldots, \mathcal{A}_n, \mathcal{A}_{n+1}]$ *is an argumentation line for* $\mathcal{A}_1$*.*

The dialectical tree *for* $\mathcal{A}_1$ *is the set of maximal argumentation lines rooted in* $\mathcal{A}_1$ *arranged in the form of a tree, and is denoted* $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A}_1)$ *(see [13] for more details). The bottom-up* marking *procedure of a dialectical tree* $\mathcal{T}$ *is as follows:*

(1) mark all terminal nodes of $\mathcal{T}$ with a U (for undefeated);
(2) mark a node $\mathcal{B}$ with a D (for defeated) if it has a children node marked U;
(3) mark $\mathcal{B}$ with U if all its children nodes are marked D.

Finally, the notion of warranted literal is defined as follows. Notice that in the presence of weights it makes sense to actually consider two notions of warrant, the usual one and another one witnessing the highest weight with which a literal can be warranted, see e.g. [3,8].

Definition 14 (Warrant, Strong Warrant). *Given a* pt-DeLP *program (Π, Δ) and a query temporal literal (ℓ, t), we say (ℓ, t) is warranted in (Π, Δ), denoted $(\ell, t) \in \mathsf{warr}(\Pi, \Delta)$, if, for some $r > 0$, there exists an argument $\mathcal{A}$ for $(\ell, t)_r$ such that $\mathcal{A}$ is undefeated in $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$. In case there is no value $s > r$ such that $(\ell, t)_s$ is also warranted, we say that $(\ell, t)_r$ is strongly warranted in (Π, Δ), denoted $(\ell, t)_r \in \mathsf{swarr}(\Pi, \Delta)$.*

It should be noted that, as expected, the set $\mathsf{warr}(\Pi, \Delta)$ is always consistent, even with the $(\tau \times \pi)$ defeat relation, but its consistency with the strict part of the program Π cannot be guaranteed.

4 Relating pt-DeLP to t-DeLP and P-DeLP

Now we proceed to study how pt-DeLP relates to each of the frameworks t-DeLP and P-DeLP. To this end, we first propose a translation between the respective languages.

Definition 15. *We define the translation maps f and g from, respectively, the language of* t-DeLP *and* P-DeLP *into that of* pt-DeLP*. These maps are the following:*

$f:$	t-DeLP	$\longmapsto$	pt-DeLP
fact	(ℓ, t)	$\longmapsto$	$(\ell, t)_1$
rule	$(\ell, t) \longleftarrow (\ell_1, t_1), \ldots, (\ell_n, t_n)$	$\longmapsto$	$\langle (\ell, t) \longleftarrow (\ell_1, t_1), \ldots, (\ell_n, t_n) \rangle_r$
			where $\begin{cases} r = 1 & \text{if } \delta \in \Pi \\ r = .5 & \text{if } \delta \in \Delta \end{cases}$
$g:$	P-DeLP	$\longmapsto$	pt-DeLP
fact	$(\ell)_r$	$\longmapsto$	$(\ell, 0)_r$
rule	$\langle \ell \longleftarrow \ell_1, \ldots, \ell_n \rangle_r$	$\longmapsto$	$\langle (\ell, 0) \longleftarrow (\ell_1, 0), \ldots, (\ell_n, 0) \rangle_r$

If P is a set of t-DeLP formulas, we will denote its translation by f by $f[P]$ or simply fP, and analogously with g. Note that both mappings f and g preserve the strict or defeasible character of facts and rules (in the case of g the weight is also preserved).

Lemma 1. *For arbitrary arguments $\mathcal{A}, \mathcal{B}$ in a* t-DeLP *program P, we have $\mathcal{A} \succ_\tau \mathcal{B}$ iff $f[\mathcal{A}] \succ_\tau f[\mathcal{B}]$ iff $f[\mathcal{A}] \succ_{\tau\times\pi} f[\mathcal{B}]$ iff $f[\mathcal{A}] \succ_{\pi\times\tau} f[\mathcal{B}]$. The case of $\preceq\succeq_\tau$, $\preceq\succeq_{\tau\times\pi}$ and $\preceq\succeq_{\pi\times\tau}$ is similar.*

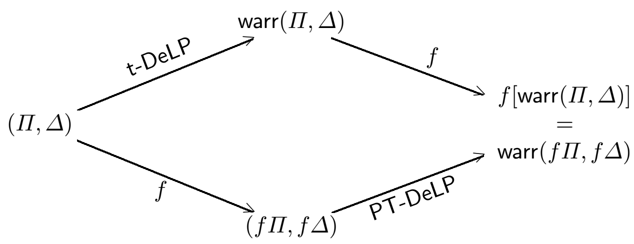

Fig. 1. A representation of pt-DeLP being a conservative extension of t-DeLP under the translation map f, and for any of the two criteria $(\tau \times \pi)$ and $(\pi \times \tau)$

Proof. (Sketch) The idea is to use the fact that conflicting arguments are nonetheless consistent with Π, so they must both be of degree .5. This makes the degrees collapse and hence makes the $\preceq\succeq_\pi$ always true. Thus, the result is determined purely by $\succ_\tau$ and $\preceq\succeq_\tau$ relations.

Lemma 1 fails for the mapping g between P-DeLP and pt-DeLP, as shown in Example 1 below. From now on, the results focus then on the relation between t-DeLP and pt-DeLP. Indeed, if we ignore the notational differences between

– t-DeLP (being strictly vs. defeasibly derivable), and
– pt-DeLP with set of degrees $\{0, .5, 1\}$ (i.e. being derivable with degree .5 vs. with degree 1)

we can define the conditions for pt-DeLP to be a conservative extension of t-DeLP as follows: *the* warr$(\cdot,\cdot)$ *operator commutes with* f; see Figure 1.

Proposition 1. *pt-DeLP is a conservative extension of t-DeLP, under both lexicographic orderings* $(\tau \times \pi)$ *or* $(\pi \times \tau)$.

Proof. (Sketch) The proof proceeds by using Lemma 1 and showing by induction that the dialectical trees $\mathcal{T}_{(\Pi,\Delta)}(\mathcal{A})$ and $\mathcal{T}_{(f\Pi,f\Delta)}(f[\mathcal{A}])$ are isomorphic.

In contrast to this preservation result for t-DeLP and pt-DeLP, the mapping g fails to guarantee the warrant literals from P-DeLP to pt-DeLP for both lexicographic criteria, as the following example shows.

Example 1. Consider the P-DeLP-program (Π, Δ) containing only the literals and rules mentioned in the arguments below.

$$\mathcal{A} = \{(p)_1; \langle q \longleftarrow p\rangle_{.8}\} \text{ with conclusion } (q)_{.8}$$
$$\mathcal{B} = \{(p)_1; (r)_1; \langle \neg q \longleftarrow p, r\rangle_{.8}\} \text{ with conclusion } (\neg q)_{.8}$$

Clearly, $\mathcal{A}$ and $\mathcal{B}$ attack each other, and are blocking defeaters for each other. The dialectical tree for $\mathcal{A}$ is $[\mathcal{A}, \mathcal{B}]$ which makes $\mathcal{A}$ defeated. Since this is the only argument for $(q)_{.8}$, we have $(q)_{.8} \notin$ warr(Π, Δ). The tree for $\mathcal{B}$ is $[\mathcal{B}, \mathcal{A}]$, and we also have that $(\neg q)_{.8} \notin$ warr(Π, Δ). On the other hand, if we translate them into pt-DeLP as $\mathcal{A}' = g[\mathcal{A}]$ and $\mathcal{B}' = g[\mathcal{B}]$, we find that:

1. Case $(\tau \times \pi)$: From $\mathcal{B}' \succ_\tau \mathcal{A}'$, we infer $\mathcal{B}' \succ_{\tau\times\pi} \mathcal{A}'$, so $\mathcal{T}_{(g\Pi,g\Delta)}(\mathcal{B}') = [\mathcal{B}']$ and $g(\mathsf{concl}(\mathcal{B})) = (\neg q, 0)_{.8} \in \mathsf{warr}(g\Pi, g\Delta)$.
2. Case $(\pi \times \tau)$: Since $\mathcal{A}' \preceq\succeq_\pi \mathcal{B}'$, we try the temporal criteria which as before gives $\mathcal{B}' \succ_{\pi\times\tau} \mathcal{A}'$ from $\mathcal{B}' \succ_\tau \mathcal{A}'$. As before, this makes $\mathcal{B}'$ undefeated and $\mathcal{A}'$ defeated (in the corresponding trees), so $(\neg q, 0)_{.8} \in \mathsf{warr}(g\Pi, g\Delta)$.

Thus, for both criteria, pt-DeLP is not a conservative extension of P-DeLP.

5 An Example

Consider the following scenario:

Lars, a tourist visiting the Snake Forest, just got bitten *by a poisonous snake. Normally, the bite of this kind of snake increases the likelihood of* fast poisoning*, whose maximum degree is reached 10 hours after the bite. On the other hand, Lars (who, as an experienced tropical tourist, has been bitten a few times before) has developed some resistance to the poison. So this poison increases instead his likelihood of* slow poisoning*, the maximum degree of likelihood for this being reached 20 hours after the bite. In any case, once bitten, the actual occurrence of slow or fast poisoning causes* death *immediately. Assuming Lars arrives alive to the* hospital *and he is given an antidote, the likelihood that Lars survives the next k hours depends on how much time passed between the bite and the antidote.*

So we decide to take Lars to the nearest hospital, which normally takes 8 hours. But, the radio just announced we will find a traffic jam*, causing a delay of 4 more hours. Thus, most plausibly it will take us 12 hours to reach the hospital. The problem is then to compute at time* 0 *(now) how strong is our belief of whether Lars will die (or not) at some later time t in this scenario.*

This example extends a similar scenario considered in [17], where only the amount and relevance of temporal information was considered. The possibilistic degrees allow for more smooth descriptions of the causal relations considered.

The scenario is formalized in Figure 2. The strict facts denote the factual knowledge about the initial state (where the reasoning takes place). The strict rules describe the effect of two possible ways in which the bite can lead to death by poisoning (one way would be faster than the other). In either case, the immediate effect is death. A faster poisoning does occur in normal people, while certain resistance can be acquired with experience, giving some extra temporary resistance (slow poisoning). The defeasible rules δ_2, δ_3 describe the likelihood of, resp., faster and slower poisoning across time, provided Lars has been bitten (resp. and experienced). This likelihood is represented in Figure 3 (Top). The X-axis represents time (from now, 0, to 20 hours into the future). The Y-axis describes the degree for derivability of slow and faster poisoning (i.e. death) at each time t. The arguments built in this program are also shown in Figure 4. The first and third arguments are those built from δ_2 (in grey, denoting defeated) and δ_3 (white, denoting undefeated). In order to prevent conclusions that are too pessimistic (about Lars' chances) using δ_2, a canceling rule δ_4 is introduced. So the argument for δ_3 can impose the (correct) degree $\alpha_3(\mathsf{d}_0)(= \alpha_3(t-0) = \alpha_3(t))$,

Π − facts

$$\left\{\begin{array}{ll}(\mathsf{@forest}(\text{Lars}),0)_1, & (\mathsf{bitten}(\text{Lars}),0)_1, \\ (\neg\mathsf{dead}(\text{Lars}),0)_1, & (\mathsf{exp}(\text{Lars}),0)_1\end{array}\right\}$$

Π − rules

$\langle \mathsf{dead}(\text{Lars}) \longleftarrow \mathsf{fast.poisoning}(\text{Lars}),0)\rangle_1$	δ_0
$\langle \mathsf{dead}(\text{Lars}) \longleftarrow \mathsf{slow.poisoning}(\text{Lars}),0)\rangle_1$	δ_1

Δ

$\langle \mathsf{fast.poisoning}(\text{Lars}) \longleftarrow ((\mathsf{bitten}(\text{Lars}),\mathtt{d}_0))\rangle_{\alpha_2(\mathtt{d}_0)}$ δ_2

where $\alpha_2(\mathtt{d}_0)$ is as in Fig. 3(Top).

$\langle \mathsf{slow.poisoning}(\text{Lars}) \longleftarrow ((\mathsf{bitten}(\text{Lars}),\mathtt{d}_0)),(\mathsf{exp}(\text{Lars}),\mathtt{d}_1)\rangle_{\alpha_3(\mathtt{d}_0)}$ δ_3

where $\alpha_3(\mathtt{d}_0)$ is as in Fig. 3(Top).

$\langle \neg\mathsf{fast.poisoning}(\text{Lars}) \longleftarrow ((\mathsf{bitten}(\text{Lars}),\mathtt{d}_0)),(\mathsf{exp}(\text{Lars}),\mathtt{d}_1)\rangle_{\alpha_4(\mathtt{d}_0)}$ δ_4

where $\alpha_4(\mathtt{d}_0) = \alpha_3(\mathtt{d}_0) + .001$

$\langle \mathsf{@hospital}(\text{Lars}) \longleftarrow (\mathsf{bitten}(\text{Lars}),\mathtt{d}_0),(\mathsf{@forest}(\text{Lars}),\mathtt{d}_1)\rangle_{\alpha_5(\mathtt{d}_1)}$ δ_5

$$\text{where } \alpha_5(\mathtt{d}_1) = \begin{cases} .9 & \text{if } \mathtt{d}_1 = 8 \\ 0 & \text{otherwise} \end{cases}$$

$\langle \neg\mathsf{@hospital}(\text{Lars}) \longleftarrow (\mathsf{bitten}(\text{Lars}),\mathtt{d}_0),(\mathsf{@forest}(\text{Lars}),\mathtt{d}_1),$
$(\mathsf{traffic.jam},\mathtt{d}_2))\rangle_{\alpha_6(\mathtt{d}_1)}$ δ_6

$$\text{where } \alpha_6(\mathtt{d}_1,\mathtt{d}_2) = \begin{cases} .95 & \text{if } \mathtt{d}_1 = 8 = \mathtt{d}_2 \\ 0 & \text{otherwise} \end{cases}$$

$\langle \neg\mathsf{slow.poisoning}(\text{Lars}) \longleftarrow (\mathsf{bitten}(\text{Lars}),\mathtt{d}_0),\mathsf{@hospital}(\text{Lars}),\mathtt{d}_1),(\mathsf{exp}(\text{Lars}),\mathtt{d}_2))\rangle_{\alpha_7}$ δ_7

$$\text{where } \alpha_7(\mathtt{d}_0,\mathtt{d}_1) = \begin{cases} \alpha_3(\mathtt{d}_0) + .001 & \text{if } \mathtt{d}_1 < \mathtt{d}_0 \\ 0 & \text{otherwise} \end{cases}$$

$\langle \mathsf{@hospital}(\text{Lars}) \longleftarrow (\mathsf{bitten}(\text{Lars}),\mathtt{d}_0),(\mathsf{@forest}(\text{Lars}),\mathtt{d}_1),$
$(\mathsf{traffic.jam},\mathtt{d}_2))\rangle_{\alpha_6(\mathtt{d}_1,\mathtt{d}_2)}$ δ_8

$$\text{where } \alpha_6(\mathtt{d}_1,\mathtt{d}_2) = \begin{cases} .95 & \text{if } \mathtt{d}_1 = 12 = \mathtt{d}_2 \\ 0 & \text{otherwise} \end{cases}$$

$\langle(\mathsf{dead}(\text{Lars}) \longleftarrow (\mathsf{bitten}(\text{Lars}),\mathtt{d}_0),(\mathsf{@hospital}(\text{Lars}),\mathtt{d}_1),$
$(\mathsf{exp}(\text{Lars},\mathtt{d}_2))\rangle_{\alpha_9(\mathtt{d}_0,\mathtt{d}_1)}$ δ_9

$$\text{where } \alpha_9(\mathtt{d}_0,\mathtt{d}_1) = \begin{cases} \alpha_3(\mathtt{d}_1 - |\mathtt{d}_0 - \mathtt{d}_1|) & \text{if } \mathtt{d}_1 < \mathtt{d}_0 \\ 0 & \text{otherwise} \end{cases}$$

Fig. 2. The list of strict facts and rules, and defeasible rules for the example

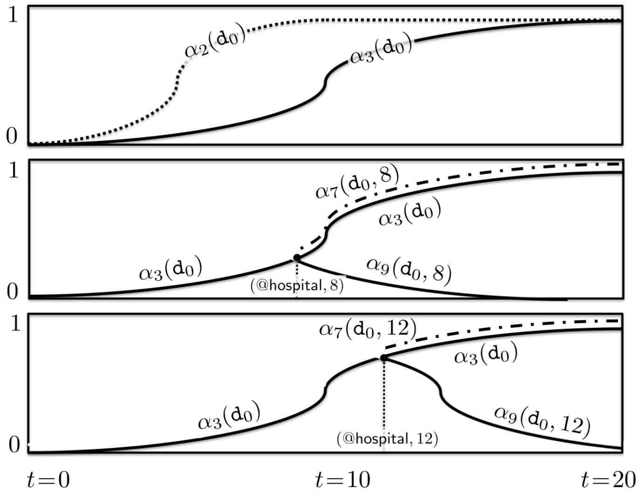

Fig. 3. The Snake-Bites-Lars example. (Top) A comparison between the likelihood of fast-poisoning (resp. slow-poisoning) in dashed arrow (resp. solid arrow). (Mid) The likelihood of Lars dying at t, if we reach the hospital at time 8; this is described by α_3 -before reaching the hospital-, and by α_9 -afterwards. (Bottom) Similarly for the case we reach the hospital at time 12, which is the actual case.

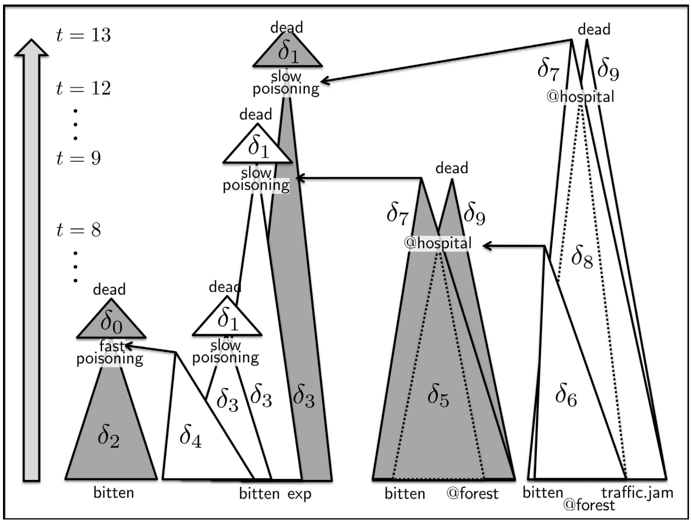

Fig. 4. An illustration of the example from Section 5 in terms of defeaters. Triangles represent arguments, with the conclusion on top, and the base on bottom. White triangles are undefeated arguments, grey arguments are defeated. The arrows denote the defeat relations. Conclusions attacking their negations are left blank. For both criteria $(\tau \times \pi)$ and $(\pi \times \tau)$, the output $\mathsf{swarr}(\Pi, \Delta)$ includes $(\mathsf{dead}(\text{Lars}), t)_{\alpha_3}$ for $t < 12$ and $(\mathsf{dead}(\text{Lars}), t)_{\alpha_9}$, for $t \geq 12$, as well as $(@\mathsf{hospital}(\text{Lars}), t)_{.95}$.

correct at least up to the point where we reach the hospital. See in Figure 4 how the counter-argument based on δ_4 acts against δ_2 at a particular time point, to let δ_3-arguments succeed.

After we reach the hospital, the previously correct α_3 becomes too pessimistic, since it does not take into account the antidote, and prevents the correct degrees to surface (the lower degrees described by α_9). To this end, δ_3-based arguments are to be counter-argued by arguments with δ_7. Now, arguments containing δ_9 or δ_7 depend of course on the time t we reach the hospital: we have arguments for $t = 8$ and for $t = 12$, though the latter is better supported. See Figure 3 (Mid) for the $t = 8$ case and (Bottom) for the (correct) case $t = 12$. Thus, the output $\mathsf{swarr}(\Pi, \Delta)$ includes $(\mathsf{dead}(\text{Lars}), t)_{\alpha_3(t)}$ for $t < 12$ and $(\mathsf{dead}(\text{Lars}), t)_{\alpha_9(t)}$, for $t \geq 12$, as well as $(@\mathsf{hospital}(\text{Lars}), t)_{.95}$. These output results from either criteria $(\tau \times \pi)$ and $(\pi \times \tau)$. So we could tell Lars that he will not be in much danger most of the time.

6 Related Work and Conclusions

We presented a new argumentation-based logic programming formalism that combines previous work on temporal reasoning, on the one side, and possibilistic reasoning on the other. This is done by taking the lexicographic product of the defeat criteria of these two systems. We have studied two notions of warrant (with and without degrees). As a result, we have shown that the combined system extend the temporal defeasible framework but not the possibilistic one.

Many different approaches exist in the area of temporal reasoning. Within the more specific field of defeasible logics (for non-monotonic reasoning), one can point to the initially proposed rule-based systems [6,16], recently extended in [14]. Since [12], though, much work has been done on argumentation-based systems, for deliberative agents who have reasons for and against claims. Most works in this area that address temporal argumentation, though, do so by associating time intervals to literals and arguments [15,5,9]. The present pointwise approach, based on [17], makes t-DeLP (or pt-DeLP) simpler than these while keeping enough expressive power. On the other hand, several works on possibilistic defeasible logic programming can be mentioned, e.g. [2,3,8]. We have assumed part of this work in the present possibilistic temporal approach. Although as far as we know, combining possibilistic and temporal defeasible argumentation is novel, the issue of combining time and possibilistic logic had already been addressed in [10].

As for future work, we would consider the *rationality postulates* of [7], which specify reasonable constraints on the logical behavior of the warrant operator. We expect that some partial solution to this problem would come by trying to extend the results for the temporal case in [17]. A more radical option would be to adopt a recursive semantics (instead of the current based on dialectical trees) like in [1] that would ensure the indirect consistency postulate.

Acknowledgments. The authors are grateful to the anonymous reviewers for their insightful comments. They also acknowledge partial support by the Spanish projects *Agreement Technologies* (CONSOLIDER CSD2007-0022, INGENIO

2010), ARINF (TIN2009-14704-C03-03) and TASSAT (TIN2010- 20967-C04-01), as well as by the Marie Curie IRSES Project MaToMuVI (FP7-PEOPLE-2009).

References

1. Alsinet, T., Béjar, R., Godo, L.: A characterization of collective conflict for defeasible argumentation. In: Proc. of COMMA 2010, pp. 27–38 (2010)
2. Alsinet, T., Chesñevar, C.I., Godo, L., Sandri, S., Simari, G.R.: Formalizing argumentative reasoning in a possibilistic logic programming setting with fuzzy unification. Int. J. Approx. Reasoning 48(3), 711–729 (2008)
3. Alsinet, T., Chesñevar, C.I., Godo, L., Simari, G.R.: A Logic Programming Framework for Possibilistic Argumentation: Formalization and Logical Properties. Fuzzy Sets and Systems 159, 1208–1228 (2008)
4. Andréka, H., Ryan, M., Schobbens, P.Y.: Operators and laws for combining preference relations. J. of Logic and Computation 12(1), 13–53 (2002)
5. Augusto, J.: Simari G. R. Temporal Defeasible Reasoning. Knowledge and Information Systems 3, 287–318 (2001)
6. Billington, D.: Defeasible logic is stable. J. of Logic and Computation 3, 379–400 (1993)
7. Caminada, M., Amgoud, L.: On the evaluation of argumentation formalisms. Artificial Intelligence 171, 286–310 (2007)
8. Capobianco, M., Simari, G.R.: A Proposal for Making Argumentation Computationally Capable of Handling Large Repositories of Uncertain Data. In: Godo, L., Pugliese, A. (eds.) SUM 2009. LNCS, vol. 5785, pp. 95–110. Springer, Heidelberg (2009)
9. Cobo, L., Martínez, D., Simari, G.R.: On Admissibility in Timed Abstract Argumentation Frameworks. In: Proc. of ECAI 2010, pp. 1007–1008. IOS Press (2010)
10. Dubois, D., Lang, J., Prade, H.: Timed possibilistic logic. Fundamenta Informaticae 15(3-4), 211–234 (1991)
11. Dubois, D., Prade, H.: Possibilistic logic: a retrospective and prospective view. Fuzzy Sets and Systems 144(1), 3–23 (2004)
12. Dung, P.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence 77(2), 321–357 (1995)
13. García, A., Simari, G.R.: Defeasible logic programming: An argumentative approach. Theory and Practice of Logic Programming 4(1+2), 95–138 (2004)
14. Governatori, G., Terenziani, P.: Temporal Extensions to Defeasible Logic. In: Orgun, M.A., Thornton, J. (eds.) AI 2007. LNCS (LNAI), vol. 4830, pp. 476–485. Springer, Heidelberg (2007)
15. Mann, N., Hunter, A.: Argumentation Using Temporal Knowledge. In: Proc. of COMMA 2008, pp. 204–215. IOS Press (2008)
16. Nute, D.: Defeasible logic. In: Handbook of Logic in Artificial Intelligence and Logic Programming, vol. 3, pp. 353–395. Oxford University Press (1994)
17. Pardo, P., Godo, L.: t-DeLP: A Temporal Extension of the Defeasible Logic Programming Argumentative Framework. In: Benferhat, S., Grant, J. (eds.) SUM 2011. LNCS, vol. 6929, pp. 489–503. Springer, Heidelberg (2011)