

Robust coordination in large convention spaces

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Abstract. Regulating the behavior of autonomous agents is necessary to solve coordination problems and minimize conflicts in multi-agent systems (MAS). Social conventions can be regarded as coordination schemes that can be employed by agents to successfully coordinate. However, to have agents agree on good conventions, without the need of a central authority, is a challenging issue. In this paper we design a novel spreading-based convention emergence mechanism that helps agents distributedly agree on the best convention when there are multiple alternatives. We apply our convention emergence mechanism to a problem with a large convention space: finding a common vocabulary (lexicon) for the agents of a MAS that allows them to perfectly communicate with neither ambiguity nor inconsistencies. Thus, we empirically show the scalability of our approach in large (in terms of agents and conventions) scenarios that change over time. Moreover, since communication is crucial to spreading, we also show that our proposed spreading mechanism is resilient to unreliable communications, thus guaranteeing the robust emergence of conventions.

Keywords: Multi-agent coordination, convention emergence, robustness

1. Introduction

Regulating the behavior of autonomous agents in multi-agent systems (MAS) to improve their overall performance and effectiveness is important to solve coordination problems and minimize conflicts. Since centralized techniques employing global knowledge are not viable, distributed mechanisms that help agents coordinate through social conventions are particularly valuable.

Spreading-based approaches have proved to be able to establish conventions in agent populations [9,22]. Spreading is a natural phenomenon whose objective is to propagate some characteristics (e.g., opinion, belief, cultural trait) over the members of a population to prompt a significant number of them to adopt it. There is a wealth of examples of spreading in *Nature* (e.g., contagion of infectious diseases, computer viruses, gossips) and their dynamics have been analyzed in different fields, such as epidemiology [16], statistical mechanics [20] and social sciences [18]. We observe that spreading-based mechanisms are promising to help agents rapidly agree on some convention(s). However,

these spreading-based models, mainly based on epidemiology, are lacking to realize the emergence of complex conventions because they limit themselves to analyzing how a *single* characteristic spreads through an agent population. Each population member may take one out of two possible states: either the characteristic is present or not (e.g., infected vs. susceptible, opinion vs. no opinion). This is not enough for MAS since a number of further issues must be considered.

Firstly, typically there will be a space of multiple convention alternatives (the so-called *convention space* [10]) from which agents make a collective choice. Thus, multiple convention alternatives (rather than two), here on referred as *convention seeds*, will be competing with each other to spread through an agent population. Moreover, in general not all the available conventions are equally preferred, because some of them support coordination more effectively. Therefore, a convention emergence mechanism must help select the best convention(s). Secondly, there is no guarantee that the good convention seeds are known (to be spread) by any of the agents in the population. Therefore, an emergence mechanism must allow agents to build (search for) new convention seeds if needed. Finally, since spreading relies on propagating

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information, if agents' communications become unreliable (e.g., by noise, maliciousness, errors), convention emergence may fail. Hence, an emergence mechanism must be resilient to unreliable propagations.

In this paper we extend the convergence emergence mechanism detailed in [23,24] to design a novel, robust spreading-based convention emergence mechanism that helps agents agree on the *best* convention(s) for coordination under the assumptions that: (i) there are *multiple* convention alternatives and some of them are preferred to others; and (ii) unreliable communications may jeopardize the spreading of conventions. Therefore, our spreading-based mechanism guarantees the robust emergence of conventions. To the best of our knowledge, no convention emergence mechanism in the literature has addressed all these requirements so far as we do in this paper.

To validate our robust spreading-based mechanism, we have selected language coordination as our case study domain because: (i) it provides large convention spaces with multiple, alternative conventions; and (ii) it is a relevant problem for MAS. In MAS, communication is a key factor for agents to successfully interact with each other. In particular, when agents rely on explicit communication, a *shared language or vocabulary* (i.e., communication system) is highly necessary. Nevertheless, in open, heterogeneous MAS, where no central authority exists, such language may not exist. Since no one enforces a common language, agents may have their own, limiting their successful interactions to agents with a similar or the same language (if any exists). In such MAS, agents may use different terms to refer to the same concept, or may use the same term to refer to different concepts, creating ambiguities in their communications. Therefore, a mechanism that allows agents to distributedly reach language conventions (consensus) that improve their communications is necessary. Furthermore, in an open MAS, establishing conventions with an *off-line* process may not be reliable. Because, the MAS conditions can change with time (e.g., the number of agents, their objectives, the environment). Hence, the need for a mechanism that allows agents to reach language conventions at the same time they normally operate to achieve their (individual) goals.

We perform an empirical analysis of our convention emergence mechanism along two directions.

Firstly, we show that our mechanism guarantees the emergence of a lexicon in large (in terms of agents and convention alternatives) scenarios and under different communication topologies. More precisely, we show

that the lexicon that emerges, agents agree upon, is a so-called *perfect communication system* [8]. Since several studies show that the social structure of a population affects how a language emerges [6,13,21], we further explore how different complex networks, such as small-world [33] and scale-free [1], as underlying topologies of our MAS may influence the results.

Secondly, we analyze the robustness of our approach in dynamic settings by: (i) allowing new agents to join a MAS at any time and hence changing its interaction topology; and (ii) considering highly unreliable information being exchanged between agents. We observe that our convention emergence mechanism is highly resilient to both changes in the network and the harmful effects of unreliable information.

The paper is organized as follows. Section 2 reviews the relevant literature. Section 3 briefly describes the illustrative domain. Section 4 presents in detail our spreading mechanism. Section 5 empirically analyzes our spreading mechanism in different scenarios. Finally, in Section 6 we draw some conclusions and set paths to future research.

2. Related work

Coordination is an important topic in AI since many large-scale applications are formulated in terms of spatially or functionally distributed entities. Coordination enables the different entities to work more efficiently and to complete activities they cannot accomplish individually. As argued in [27] coordination can be achieved via social laws and conventions. Social conventions help balance agents' individual goals with respect to those of the society so that each agent can pursue its individual goals without preventing other agents from pursuing theirs. Sociologically speaking, a convention results when members of a population adhere to some behavior, which is neither dictated nor enforced by a central authority. Therefore, there is the challenging issue of "how conventions can emerge dynamically as a result of a learning process within the agent population" (*sic*) [27], as researched in [14,28,32].

On the one hand, spreading-based approaches have proved to be able to establish conventions in agent populations [9,22]. Typically, a spreading mechanism encompasses some spreading (information transfer) strategy (on the sender side) along with some selection strategy for incoming transfers (on the receiver side). The simplest information transfer strategy proposed

in the literature is *copy-transfer* [4]: each agent tries to copy its complete convention to its neighboring agents. Copy attempts are locally controlled by some *spreading rate*, a parameter that governs the spreading pressure through the population [20]. As to selection, the simplest strategy is *random*: each agent randomly accepts one out of all its incoming transfers (conventions). However, as argued above, current spreading-based models in the literature, mainly based on epidemiology, are lacking to realize the emergence of conventions because they limit themselves to analyzing how a single characteristic spreads through an agent population. Each population member may take one out of two possible states: either the characteristic is present or not (e.g., infected vs. susceptible, opinion vs. no opinion).

On the other hand, evolutionary algorithms (EA) have been employed as means of establishing conventions either: as a centralized process [12]; or as an individual process embedded in each agent [19]. In [12] (the centralized approach), a global EA tries to find a set of rules (convention) that govern the behavior of all agents. This is achieved by maintaining multiple societies each with a different convention and then applying the evolutionary process to them. Such approach presents some disadvantages. First of all, it is an *off-line* process, i.e., the algorithm is run for mock-up systems and when the best conventions are found they are hardwired on agents. Additionally, as the complexity of the domain grows this approach becomes very slow because it requires multiple simulations at the same time. Furthermore, it is a top-down approach, whereas the approach proposed in this paper is a bottom-up method (namely, agents reaching conventions by themselves).

Alternatively, [19] proposes to embed an EA in each agent so that each agent employs some reinforcement learning to find its appropriate behavior. These methodologies do not have an explicit propagation mechanism, though conventions still arise. Agents that interact frequently with each other must evolve policies that decrease conflicts among them. The problem with this approach is that the time required for all the agents to stabilize their EA may be too long and then it is also an *off-line* methodology.

The spreading mechanism proposed in this paper extends a basic spreading mechanism along several directions by incorporating some EA principles. Firstly, based on the principle behind the recombination technique from the evolutionary algorithms literature [3], our spreading mechanism employs a *partial-transfer*

(instead of copy-transfer) spreading strategy: an agent receiving a partial convention can combine it with its own convention to create a new one. This allows agents to explore a space of multiple conventions as well as discovering conventions that are initially unknown by the agent population. Secondly, drawing inspiration from the principles of replicator dynamics [34], our spreading mechanism employs *elitist* (instead of random) selection as selection strategy. Thus, an agent chooses the incoming convention with the highest valuation whenever this is higher than its own convention. This selection strategy allows to look for conventions that are more preferred than others. Besides the spreading and selection components, our spreading mechanism employs further components. On the one hand, we employ innovation, *controllable* internal noise, to enhance the exploration capabilities of the spreading mechanism, leading the agent population to the best convention. This interest is motivated by the observation in the literature (e.g., [18]) that noise can be either beneficial or else lead to chaos. On the other hand, each agent incorporates a *self-protection* component that locally and dynamically controls the acceptance of incoming transfers. By dynamically changing between acceptance and rejection of incoming transfers, each agent can self-protect against external noise.

To the best of our knowledge, the emergence of conventions considering the existence of multiple, possible conventions has only been very recently considered in [26]. However, this work does not consider that some conventions are more preferred than others. Regarding the emergence of conventions in dynamic (changing) environments, we are only aware of the contribution by Salazar et al. [23], though this work only considers an eight-convention space. Neither of these contributions considers dynamic environments with large convention spaces where some conventions are preferred or the presence of noise in the propagation of conventions as we consider in this paper.

Language conventionalization has been thoroughly studied in social sciences [2,15] and its properties are well known. Hence, it is a very rich scenario to validate the capabilities of spreading as a mechanism for the engineering of convention emergence. Therefore, notice that contributing to the understanding of either language emergence or evolution is beyond the scope of our work.

It has been argued that languages are established as a form of a social convention, thus the relationship between a word and a concept is dependent on the interactions between individuals. Several studies have ad-

dressed the modeling of such interactions as *language games* between individuals [5,17], with various levels of success. These games model language construction at a purely semiotic level, i.e., they neglect the semantic relationships between models and symbols. A common game to study conventionalization is the *naming game* [31]. This game focuses on the interactions of speakers and listeners that try to find names for objects to understand each other. Thus, the aim of the naming game is to study how a common lexicon (vocabulary) is established in a society.

In a broad sense, the naming game can be regarded as a model for ontology sharing [30]. Nevertheless, currently the most common formulation of the naming game presents some impractical characteristics to make it useful for open MAS. Firstly, it allows agents to create any word to refer to a particular object, which is impractical in MAS because the number of concepts to name can most likely be bounded beforehand. Secondly, it allows the existence of multiple words to refer to the very same object (synonymy), which may cause ambiguities or inconsistencies in the communication between agents. Moreover, the predominant naming game formulation makes no distinction between the communication model and the communication development (language acquisition) algorithm (i.e., they are inter-woven). To overcome these issues, in [25] Salazar et al. propose a communication model based on the one in [7]. De Jong's model borrows some of the notions of the naming game and defines them for a MAS. Moreover, it makes a distinction between the interaction model and the communication development algorithm. Nevertheless, it still considers word creation. Therefore, we propose to replace word creation by word selection (from a finite set), similar to a not commonly used variation of the naming game presented in [29].

3. Language coordination game

We shall consider an open MAS composed of autonomous agents where no central authority exists to rule the agents and that the agents may only work with local knowledge. Agents interact by exchanging messages related to certain concepts C (from a finite set) of a problem (be them, for instance, object, topics, actions) using a set of words W . The vocabulary for the messages is given by each agent's internal *lexicon*, which assigns an external representation (word) to each concept (see Fig. 1(a) for a lexicon example). Thus, the language coordination problem amounts to

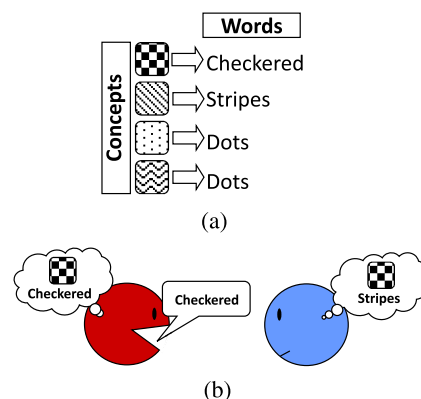


Fig. 1. Vocabulary acquisition scenario. (a) Lexicon, (b) interaction. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

find for all agents a common mapping (lexicon) that assigns a word $w \in W$ to each concept $c \in C$.

The quality of a lexicon is measured by its *specificity*. A lexicon's specificity quantifies the percentage of words that identify a single concept. Thus, a lexicon with maximum specificity (100%) represents a lexicon with one-to-one mappings. Hence, those are the most preferred lexicons by the agents because they reduce the likelihood of misunderstandings. However, a lexicon with 100% specificity is not useful if only one agent (or even only a few) use it while the rest of the agents use different ones. Therefore, a compulsory requirement for perfect communication (i.e., all agents understand each other) is that all agents agree on a common lexicon. Such lexicon convention must emerge through agents' interactions, since we assume there is no central authority to establish it.

The language coordination problem is particularly challenging. Firstly, agents' lexicons stand as convention seeds since they represent potential conventions. Hence, the number of possible conventions (the convention space) can easily become very large, depending on the number of concepts and words (W^C where W and C stand for the number of words and concepts, respectively). Furthermore, since there is no guarantee that some agent already knows a lexicon with 100% specificity at the beginning of the operation of a MAS, it is impossible (through a classic spreading approach) for agents to agree on a lexicon convention that allows perfect communication. The reason behind this is that classic spreading does not introduce new information only propagates the existing one.

We assume that interactions between agents are pairwise. Specifically in each interaction an agent (playing

a *speaker* role) uses the word assignments in its lexicon to build one-word messages, while the second agent (playing a *hearer* role) uses its lexicon to decipher the received messages. Notice that, depending on the intersection between the lexicons of the speaker and the hearer, the hearer may or may not understand the received message. For example, in Fig. 1(b), the agent on the left (the speaker) tries to convey the concept *checkered* to the agent on the right (hearer) using the word *checkered* (as given by its lexicon in Fig. 1(a)). However, since the hearer has a different concept associated to the word *checkered*, the agents do not understand each other.

Additionally, we consider that interactions between agents in a MAS are restricted by an interaction topology. We model an interaction topology as a graph (Ag, E) , where $E \subseteq Ag \times Ag$, whose edges correspond to relationships (neighborhoods) between agents. If $(ag_i, ag_j) \in E$, then ag_i and ag_j are neighbors, and thus they can interact with each other. Since the kind of MAS we consider is open (agents join or leave at will), interaction topologies may change with time.

To play a game, each agent, $ag_i \in Ag$ has a lexicon, $L_i : C \rightarrow W$, which assigns an external representation (a word) to the concepts, and a decoding function, $D_i : W \rightarrow 2^C$, which is used for translating a given word to its related concept. We restrict the lexicon in such a manner that only one entry per concept is permitted. Hence, it is not possible to assign more than one word per concept (synonymy). However, we allow polysemy¹ because it may arise while agents jointly search for a common lexicon. The mechanics of a game between two agents is the following:

1. agent ag_i selects a concept of the problem, $c \in C$;
2. agent ag_i uses its lexicon, L_i , to find the word, w , that it uses to refer to c ;
3. agent ag_i sends w to some neighbor agent ag_j ;
4. agent ag_j uses its decoding function, D_j , to interpret w into a concept $c' \in C$;
5. agent ag_j responds according to its understanding of c' ; and
6. the game is successful if agent ag_j 's response matches ag_i 's concept (i.e., if $c = c'$).

Hence, for the games to be always successful all agents must agree on a common, 100% specific lexicon. Finally, notice that the ratio between the number

of available words ($|W|$) and concepts ($|C|$) generates scenarios with different degrees of difficulty with respect to the specificity. When $|W| < |C|$ perfect communication is not possible because ambiguity is unavoidable. When $|W| = |C|$ lexicons with a 100% specificity are feasible but, for a large enough number of concepts, the resulting convention space is large ($|W|^{|C|}$) and only a small number of them present 100% of specificity. Finally, when $|W| > |C|$ many lexicons with high specificity exist and the main issue is to agree in a common one. This paper focuses on the $|W| = |C|$ case since it is the most interesting scenario (see [25] for a previous study of the $|W| > |C|$ scenario).

4. Robust convention emergence

As we have stated during this paper, our goal is to engineer the emergence of the best conventions in open, dynamic MAS. In what follows, we propose a spreading mechanism that is both distributed and adaptive so that it promotes a continuous convergence towards the best conventions despite adverse circumstances (changes in the agent population and noise). Specifically, our spreading mechanism (illustrated in Fig. 2) consists of *four components* that continuously attempt to create and spread new and better conventions. Those components are: an *information transfer* component to spread conventions to neighbors; a *selection* component aimed at selecting more promising conventions; an *innovation* component to locally enhance an agent's convention exploration; and a *self-protection* component to protect an agent against incoming unreliable information transfers. Next, we detail these components.

4.1. Information transfer

Information transfer is the component responsible for spreading convention seeds. As stated in Section 2, in its traditional form (*copy-transfer*) it simply attempts to completely replicate an agent's convention seed to its neighbors. However, such strategy clearly promotes stagnation, since at its best, the resulting convention would be the best convention seed known by the agent's during the MAS start-up.

Hence, to facilitate the creation of new convention seeds we propose the use of a *partial-transfer* strategy, which borrows the principle behind the recombination technique from evolutionary algorithms [3]. In

¹A word is associated with multiple concepts.

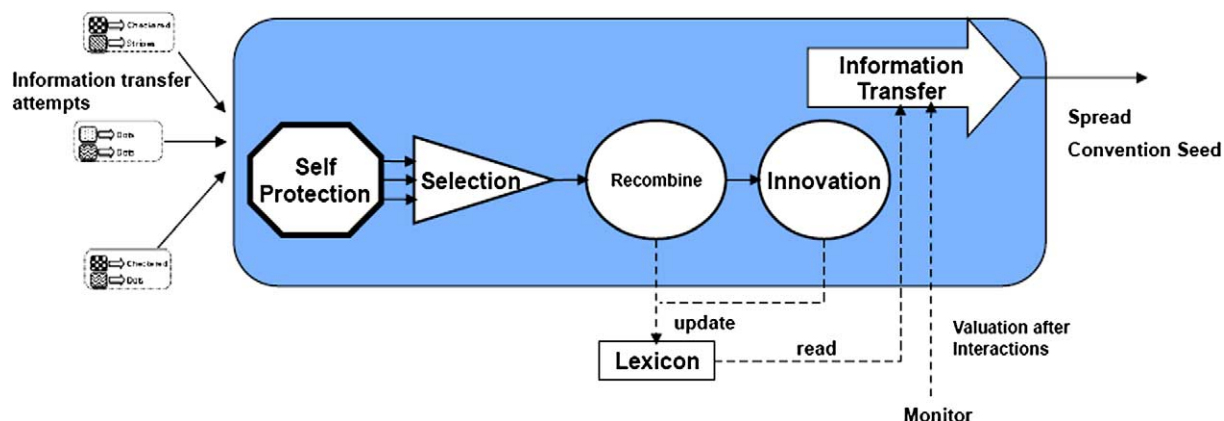


Fig. 2. A robust spreading mechanism. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

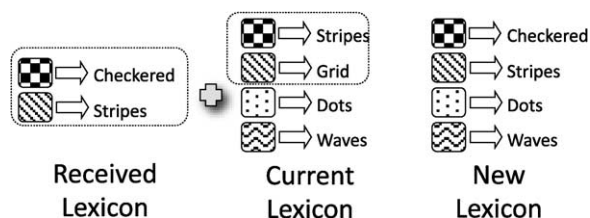


Fig. 3. Creating a new lexicon from a partial transfer.

this strategy, agents only propagate some part of their convention seeds. Thus, agents on the receiving side can create new convention seeds by updating their own with one of the incoming partial seeds. For instance, as Fig. 3 shows, an agent replaces half of its lexicon with the half it received from another agent to create a new lexicon.

Additionally, information transfer attempts are locally regulated by some *spreading rate*. This parameter serves to govern the spreading pressure through the population [20], i.e., how frequently agents attempt to promote their own conventions. Thus, agents do not actually require to constantly spread information to their neighbors (as we show in the empirical evaluation section).

In Section 5.3 we empirically show the shortcomings of a traditional copy-transfer strategy against the proposed partial-transfer strategy. Nevertheless, the experiments also illustrate that the limited exploration capabilities of partial spreading are not sufficient to (empirically) guarantee that the best convention is reached (particularly on scale-free neighborhoods).

Finally, even though information transfer is crucial for spreading, it has an inherent flaw: its sensitivity towards unreliable communications (e.g., noise). In other words, it can be adversely affected by tampering dur-

ing communications. Section 4.4 discusses this issue in more detail and proposes an approach to overcome such tamperings.

4.2. Selection

Selection is the component of the mechanism that guides convention emergence: from emerging any convention, to emerging the one that provides the best coordination (e.g., the lexicon with the highest quality).

For instance, by employing the simplest selection strategy, *random selection* (as discussed in Section 2), agents can emerge any convention regardless of its quality. However, as we have argued above not all convention seeds are equally valuable. Thus, a selection component needs to be capable of guiding the agents towards the best convention.

Nonetheless, since we assume that an agent does not know the identities of other agents² (neither during the interaction nor throughout transfers), an agent has no means of valuing the quality of its incoming convention seeds, limiting its ability to select one of them. To this end, we propose that the information transfer component includes a valuation (assessed by the sending agent) of the convention seed being spread. Therefore, each agent can use the valuations of the convention seeds it receives for selection purposes.

Taking inspiration from the principles of replicator dynamics [34] (i.e., the better the performance the more prone to replication), we propose a selection strategy (*elitist selection*) that promotes the adoption of convention seeds that can potentially improve an agent's own convention. Specifically, elitist selection

²The usual *anonymity* assumption in the emergence of conventions (see Chapter 7 in [27]).

chooses the incoming convention seed with the highest valuation if and only if such valuation is higher than an agent's valuation of its own convention seed. In other words, agents only accept incoming transfers that might improve their own convention seeds.

Section 5.3 will empirically show the shortcomings of random selection with respect to elitist selection.

4.3. Innovation

Innovation is the exploration component. Specifically, innovation is a local component that internally modifies an agent's convention seed. Thus, an agent no longer needs to depend on incoming partial transfers to create new convention seeds. For our purposes, we implement the innovation component by randomly changing the elements of a convention seed with some probability. In the language coordination game, this amounts to changing a concept-word assignment with a certain probability.

Innovation was designed as a way to internally mimic noise, since multidisciplinary studies (e.g., [18]) have shown that low noise levels can enhance exploration by introducing new traits (in our case new conventions). Hence, this component increases the likelihood of emerging the best convention for coordination.

Innovation is locally controlled by the *innovation rate*, a parameter that governs the probability of performing a random change on an agent's own convention. In Section 5.4 we empirically show how this parameter affects the emergence of the best conventions.

4.4. Self-protection

As stated in Section 4.1, unreliable communications can be an issue for spreading mechanisms because their main component, information transfer, relies on sending (propagating) information. Moreover, in open MAS unreliable communications are more prone to occur. Communications (message exchanges) between agents may be corrupted due to several causes: environmental (e.g., noisy communication channels), maliciousness (e.g., lying agents), or even as a product of some error or mistake. In general, we refer to the cause of corrupted communications as *noise*, since regardless of the source, it simply amounts to unwanted information within communications.

When considering emergence of conventions in the presence of noise, we take the stance that each agent must be able to self-protect against unreliable spreadings. Moreover, we are working under the assumption

Algorithm 1 Self-protection algorithm

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1: update bestPerformance;
2: if ( $c \geq window$  &  $stddev(performance) > threshold$ )
   then
3:   if ( $acceptance = close$ ) then
4:      $acceptance \leftarrow open$ ;
5:      $c \leftarrow 0$ ;
6:   elseif ( $performance \geq bestPerformance$ ) then
7:      $acceptance \leftarrow close$ ;
8:      $c \leftarrow 0$ ;
9:   end if
10: end if
11:  $c \leftarrow c + 1$ ;

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of anonymity (as stated in Section 4.2), which impedes us from employing the tools in the reputation literature. At this aim, we propose that each agent incorporates a *self-protection* component that locally and dynamically controls the acceptance of incoming transfers.

The self-protection component is a *local* component responsible for toggling on and off an agent's acceptance of incoming transfers (i.e., it can prevent an agent from receiving any information transfer). Thus, it distinguishes between two *acceptance states*: open and closed. The self-protection component switches between acceptance states after assessing that an agent's (communication) performance³ is not stable enough. Therefore, when an agent's acceptance state switches to closed, it temporarily rejects incoming convention seeds and instead only searches locally for better conventions.

Algorithm 1 outlines the operation of the self-protection scheme. The algorithm periodically (after some time *window* elapses) reviews an agent's acceptance state. At the end of each time window, the algorithm assesses the stability of an agent's communication performance as its variability (its standard deviation) over the window. If an agent's performance variability ($stddev(performance)$) exceeds some *threshold*, the algorithm considers that the current acceptance state (either open or closed) is not contributing to emerge (stabilize) a convention, and therefore a state switch is required. Notice that when the acceptance state is closed, the state can immediately change to open. However, an immediate switch from open to closed does not make sense because an agent might lock itself with a convention seed of bad quality. Hence, a switch from open to closed is delayed until the current performance of the agent's convention is at

³As mentioned above, the difference between successful and unsuccessful communications when acting as a speaker.

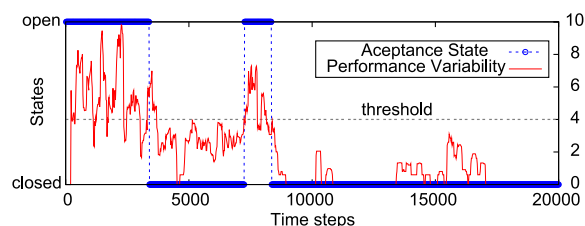


Fig. 4. The self-protection acceptance toggling from an agent's perspective on a medium noise situation. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

least as good as the best performance obtained so far (*bestPerformance*). Figure 4 depicts an agent's acceptance state toggling process. Observe that an agent's acceptance state starts as open, and even though the variability exceeds the threshold, the self-protection component cannot change the state to closed because of the risk of locking the agent with a bad convention seed. However, once a good convention (the one yielding the highest performance so far) is found, the self-protection component closes its acceptance state (~ 4000 ticks). Nonetheless, when the convention seed proves not to be stable enough it opens the state once again (~ 7000 ticks) only to close it one final time when the best convention is found.

4.5. The robust spreading mechanism

Finally, we can wrap up the local components described above to build a spreading mechanism aimed at the emergence of conventions. Notice that all the components operate at the agent level and hence they only employ local knowledge. Therefore, from each agent's point of view, the resulting spreading mechanism consists of the following operations (illustrated in Fig. 2): (1) monitor the agent's interactions to measure its performance (e.g., number of successful interactions); (2) send convention seeds to all neighbors using partial transfer; (3) determine the acceptance state (open/closed) to either accept or block the incoming conventions; (4) select (using elitist selection) an incoming seed to update its own convention seed (only when the acceptance gate is open); and (5) apply innovation to the current seed.

The robustness of the proposed mechanism results from both the innovation and self-protection components, namely by exploiting internal *controllable* noise and protecting against external *uncontrollable* noise. On the one hand, innovation helps find the best conventions when the no agent is initially aware of the best

convention seed. Additionally, it provides the means to escape from low quality conventions. On the other hand, self-protection allows the spreading mechanism to function despite highly unreliable communications.

5. Empirical evaluation

In this section we empirically show that the spreading mechanism detailed in Section 4: (i) emerges conventions in MAS with large convention spaces; and (ii) is highly robust. More precisely, we show that our spreading mechanism applied to the language coordination problem: (a) allows agents to reach lexicon convention(s) with a high level of specificity (i.e., a perfect communication system) under the most common interaction topologies; (b) allows conventions to be resilient against changes in the agent population and its underlying interaction topology; and (c) allows conventions to emerge and be maintained in spite of various levels of unreliable communications.

In Sections 5 and 5.1 we describe the interaction topologies and the empirical settings that we employed. Sections 5.3 and 5.4 show the shortcomings of traditional spreading and the effectiveness of our proposed mechanism. Next, in Section 5.5 we study a dynamic setting, where both the agent population and the interaction topology change over time, to test the robustness of our approach. Finally, in Section 5.6 we continue to validate the robustness of the mechanism by showing its effectiveness to cope with noisy communications.

5.1. Interaction topologies

It has been argued that the social distribution of individuals is an important factor in the evolution of languages [6,13,21]. This distribution is modeled in our MAS by the underlying interaction topology. Thus, in order to empirically analyze the capabilities of the spreading mechanism we chose the following interaction topologies:

Small-world networks presenting the small-world phenomenon, in which nodes have small neighborhoods and yet it is possible to reach any other node in a small number of hops. This type of networks are *highly-clustered* (i.e., have a high clustering coefficient). We note them as $W_V^{k,p}$, where V is the number of nodes, k the average connectivity, i.e., the average size of the node's neighborhood, and p the re-wiring

probability. We used the Watts and Strogatz model [33] to generate these networks.

Scale-free networks characterized by having a few nodes acting as highly-connected hubs, while the rest of them have a low connectivity degree. Scale-free networks are *low-clustered* networks. We note them as $S_V^{k,-\gamma}$, where V is the number of nodes and its degree distribution is given by $P(k) \sim k^{-\gamma}$, i.e., the probability $P(k)$ that a node in the network connects with k other nodes is roughly proportional to $k^{-\gamma}$.

5.2. Experimental settings

The large convention space of our experimental domain is given by a language coordination problem with $|C| = 10$ concepts and $|W| = 10$ words (i.e., the size of the convention space is 10^{10}). Thus, to prevent ambiguity a lexicon must manage to match each one of the ten concepts to a different word.

Each *experiment* consists of 50 discrete event simulations, each one running up to 100,000 time-steps (ticks). Each simulation runs with 1000 agents using one of the underlying topologies defined in Section 5.1. At the beginning of each simulation, each agent uploads a random lexicon. During each simulation, at each time-step, each agent interacts, through communications as defined in Section 3, with some randomly selected neighbor. The interactions occur by agents randomly choosing some concepts to send. The individual understanding of each agent, is measured every 20 ticks, as the number of successful communications, as a speaker, during that period.

We generated interaction topologies for the simulations as small-world and scale-free networks by setting the following parameters: $W_{1000}^{10,0.1}$ and $S_{1000}^{10,-3}$. The clustering coefficients of the topologies are 0.492 (highly-clustered) and 0.056 (not clustered), respectively. Notice that we generate a new interaction topology per simulation.

As for the parameters of the spreading mechanism, both values were arbitrarily chosen to be considerably low to show that even under such circumstances the proposed spreading mechanism performs well. Specifically, we chose (unless indicated otherwise) a *spreading rate* of 0.1 and an *innovation rate* of 10^{-4} .

In order to study the effect of the spreading over a MAS, we probed simulations in two ways. On the one hand, to measure whether a lexicon convention is adopted, we analyze the number of agents that share each lexicon per tick. We shall refer to the lexicon shared by the largest number of agents as the *dominant*

lexicon. On the other hand, we also analyze at every tick the *quality* of such lexicon. Given a lexicon its quality is determined by its specificity, namely the percentage of words that represent a single concept. For both dominant lexicons and specificity, we aggregate the measures obtained after 50 simulations using the inter-quartile mean.

5.3. Shortcomings of traditional spreading

In this section we empirically analyze the shortcomings of traditional spreading approaches in realistic settings. In particular, we focus on a scenario where: (1) multiple possible convention exist; and (2) the most useful (best) conventions are not known by any agent beforehand (i.e., agents are not aware of them at the operation outset of a MAS). Specifically, our experiments compare a *copy (transfer) + random (selection)* spreading mechanism (i.e., traditional spreading) against spreading mechanisms based on partial-transfer and elitist selection.

We ran experiments with different spreading rates (within $[0.1, 1.0]$) to observe: (i) if some global conventions emerged; and (ii) the quality of such conventions. On the one hand, the experiments (see squared plots in Fig. 5(a) and (c)) show that traditional spreading can indeed reach some convention, although very slowly. However, the best convention cannot be established (even with the spreading rate at its highest). Specifically, agents reach lexicons with around 70 and 63% quality on small-world and scale-free networks respectively (the squared plots in Fig. 5(b) and (d)). Regarding the effect of the topology, agents on scale-free networks begin to agree on a single convention much faster than on small-world. Scale-free topologies require, at least, 10^5 ticks, while small-world ones require, at least, 2×10^5 ticks to reach a single convention. This slower spreading diffusion exhibited on small-world facilitates the establishment of lexicons with higher quality. Since in small-world a global convention is reached at a relatively slower pace, the convention seeds with higher valuation have a better chance of being spread through all the agents.

However, by replacing random selection with elitist selection (*copy + elitist*), agents reach lexicons with 86.5% quality on small-world and 80% on scale-free (the circled plots in Fig. 5(b) and (d)). Nonetheless, it is not possible to reach a convention out of the initial agent's convention seeds. Namely, if no agent has the

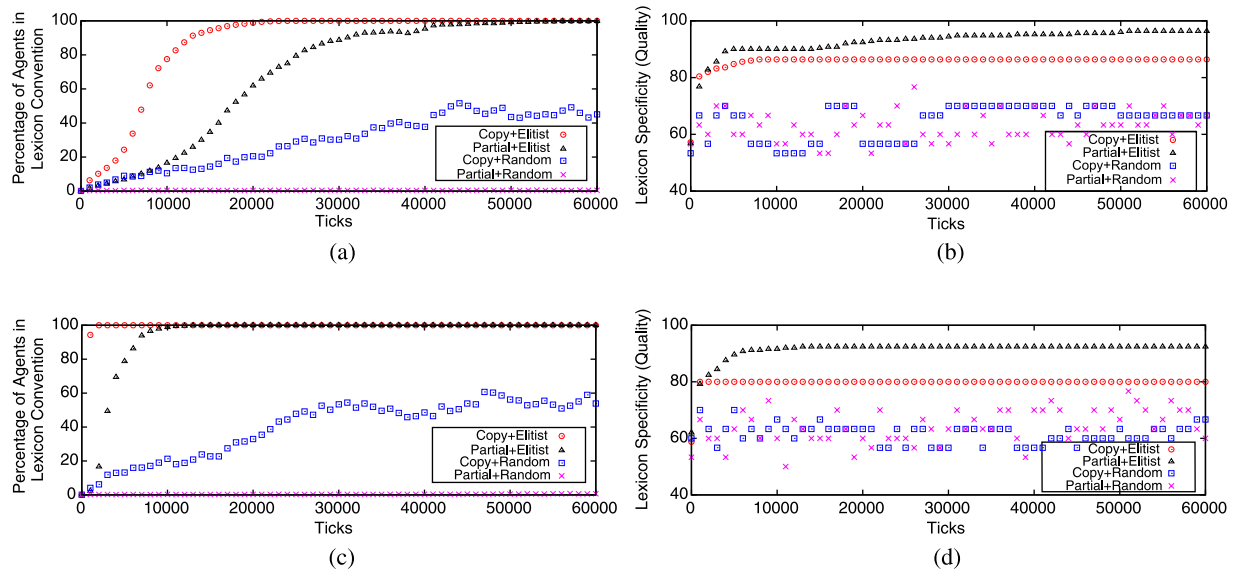


Fig. 5. Information transfer/selection combinations with low spreading rate (0.1). (a) Small-world convention agents, (b) small-world convention quality, (c) scale-free convention agents, (d) scale-free convention quality. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

best convention seed at the beginning of the MAS operation, it is not possible to spread it.

When employing *partial + elitist*, the lexicons reached had at least 90% quality on both topologies. A detailed analysis shows that for small-world topologies $\sim 70\%$ of the simulations reached a maximum quality lexicon, in contrast with $\sim 20\%$ of the scale-free simulations. Regarding convergence speed, the *partial + elitist* mechanism exhibits fast convergence, though slightly slower than *copy + elitist*. However, the value of this mechanism comes from the quality of the resulting conventions. In other words, partial-transfer benefits from its better exploration abilities to achieve higher quality conventions at the price of a slower convergence speed. Notice that global conventions are still reached much faster in scale-free networks, which limits the ability to find the best convention (there is less time to explore the convention space).

To summarize, the traditional spreading approach (*copy + random*) is not enough to deal with the features of realistic MAS settings described at the beginning of this section. Moreover, even a mechanism only composed of partial-transfer and elitist selection yields significantly better results. Nevertheless, the results are still not good enough, since a perfect communication system cannot consistently emerge. Thus, in the following subsection we evaluate a more complete mechanism.

5.4. The role of innovation

In the previous section we showed that pure spreading (be it through copy or partial transfers) has not enough exploration capabilities to reliably guide agents towards the best convention when the convention space is large. Hence, in this section we show that a spreading mechanism requires an innovation component (as described in Section 4.3) to endow it with exploration.

Small-world. The innovation component experiments show (Fig. 6(a)), that on average agents over a small-world topology exhibit a smooth growth towards a global lexicon convention (the circled plot). Such growth is similar to the one shown in Section 5.3 (i.e., innovation does not alter the convention adoption behavior). Nevertheless, now the dominant lexicon reaches maximum quality (100% specificity), i.e., a perfect communication system emerges. Notice that the quality of the lexicon improves as more agents start to share a common lexicon (the crosses plot) and when a 100% lexicon is found ($\sim 13,000$ ticks), the global convention is reached.

Moreover, observing a particular simulation provides some interesting insights. Figure 7 shows one of the 50 simulations performed for the small-world topology. In this plot, the transition towards a lexicon with 100% specificity is clearer. Observe that around 2000 ticks most of the agents ($\sim 95\%$) appear to reach

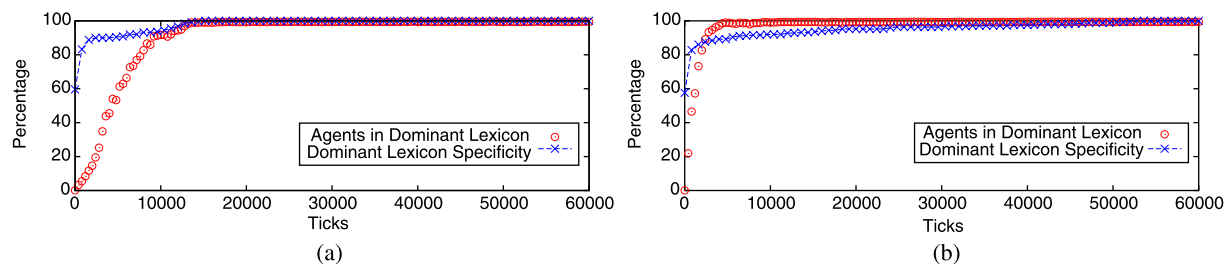


Fig. 6. Convention convergence using elitist + partial + innovation. (a) Small-world, (b) scale-free. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

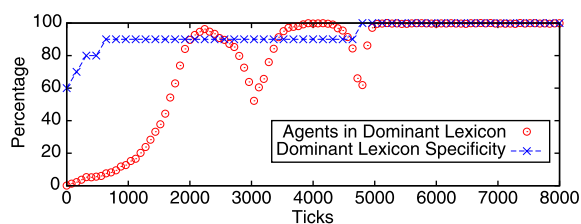


Fig. 7. Results of a particular small-world simulation. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

a convention with 90% quality. However, as a result of innovation they are promptly pulled away from that convention to another one (also with 90% quality). At around tick 4800 a lexicon with maximum quality appears and starts pulling agents and after ~ 5000 ticks the global convention settles.

Scale-free. The innovation experiments over scale-free topologies (Fig. 6(b)) reveal that a low innovation rate is enough to emerge a global lexicon with maximum quality. However, even though a global lexicon is reached quite promptly, the best lexicon requires a longer time to be found ($\sim 45,000$ ticks). In other words, the experiments once again show (as the ones in the previous section did) that such fast convergence towards a global convention can be detrimental for exploration. When most agents share a good/high quality lexicon convention, it is more difficult to sway them towards a better one (especially in games like the language coordination game, where the utility of a lexicon heavily depends on the number of agents that share it).

Therefore, although emerging perfect communication system takes time, it is still a good result. However, it is reasonable to think that we can speed up the process by increasing the innovation rate (remember that our experiments employ a considerably low value, 10^{-4}). To that end, we repeated the experiments with higher innovation rates (Table 1).

The results show that agents can indeed find a lexicon with maximum quality much faster, but at some

Table 1
Effects of innovation rates on scale-free topologies

Innovation rate	Ticks to maximum quality	Agents in convention (%)
1×10^{-4}	$\sim 45,000$	~ 99
3×10^{-4}	$\sim 21,000$	~ 94
5×10^{-4}	$\sim 18,000$	~ 90
1×10^{-3}	$\sim 15,000$	~ 81
1×10^{-2}	< 5000	< 15

cost. As the innovation rate increases, the number of agents sharing the dominant lexicon decreases. Table 1 clearly depicts this effect for different innovation rates. For instance, when using 5×10^{-4} as the innovation rate a 100% quality lexicon is found faster than using 1×10^{-4} (at $\sim 18,000$ ticks), but only 90% of the agents share the dominant lexicon. Such effect is not surprising because a higher innovation rate means that more agents change their lexicons at any point in time. Moreover, it is in line with the results in the literature (e.g., [18]) showing that a too high frequency of change can lead to chaos (e.g., the 1×10^{-2} experiments).

However, it is interesting to analyze how innovation affects spreading over scale-free topologies. From Fig. 8 we observe that with a (relatively) higher innovation rate (5×10^{-4}), the shape (plot-wise) of the dominant lexicon emergence is similar to the one presented by the small-world experiments (Fig. 6(a)), but with less agents joining the dominant convention. Therefore, the higher the innovation rate, the less successful the spreading. If this is the case, we can increase the spreading rate (so far we have been employing a low value, 0.1) to counterbalance such effect. For instance, using a higher spreading rate (0.3) with a higher innovation rate (5×10^{-4}) provides much better results (100% quality lexicon for $\sim 98\%$ of the population at $\sim 11,000$ ticks).

To summarize, experimental results show that a *partial + elitist + innovation* spreading mechanism can emerge a perfect communication system for most

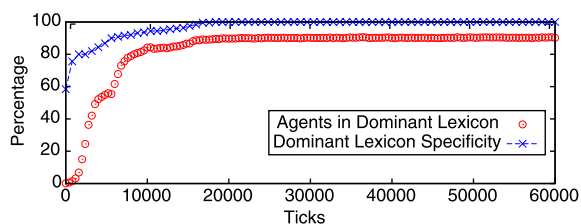


Fig. 8. Results of 5×10^{-4} innovation probability on scale-free. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

of the population regardless of the topology. However, under scale-free topologies (moderately) higher spreading and innovation rates are needed.

5.5. Dynamic population

So far we have showed the ability of our spreading mechanism to emerge conventions. Nevertheless, all scenarios considered so far were static, in the sense that the MAS did not change with time. Hence, the purpose of the next experiments is to evaluate our spreading mechanism in dynamic conditions, i.e., when the MAS changes with time.

To that end, we model the dynamics of a MAS by allowing the agent population and the agents' neighborhoods to change over time. In practice, both changes are achieved by dynamically changing the network topology. Hence, we proceeded as follows: (1) we create a scale-free network interaction topology up to a certain number of agents; (2) we let agents interact over the interaction topology; and (3) after 400 simulation ticks, we introduce 20 new agents, with random lexicons, in the agent population by wiring them to other while maintaining the properties of the scale-free (re-wiring the existing agents if necessary). We chose scale-free topologies because the Barabási–Albert (BA) scale-free network generation algorithm [1] is iterative. Hence, we can easily implement the MAS dynamics by inter-weaving the BA algorithm with the MAS simulation. In other words, we ran the MAS and the BA algorithms at the same time. The MAS employed started with a scale-free network topology with 400 agents ($S_{400}^{10,-3}$), which grew until reaching 3500 agents ($S_{3500}^{10,-3}$).

Likewise Section 5.4, agents rapidly join a common lexicon with low quality. Moreover, the incoming agents are promptly swayed towards the common lexicon convention. However, unlike the static experiments, our default low spreading and innovation rates are not enough to find the 100% specific lexicon con-

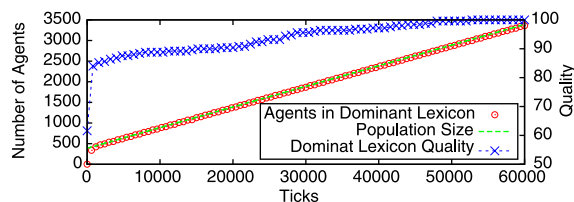


Fig. 9. Lexicon convention evolution for dynamic agent population over a scale-free topology. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

vention (at least in less than 60,000 ticks). This shows that in growing population with continuous re-wiring it is indeed challenging to emerge top quality lexicon conventions.

Nonetheless, as we learned in Section 5.4 by increasing the innovation rate agents become more prone to find the best lexicon convention. Figure 9 illustrates the results of employing a moderately higher innovation rate (3×10^{-4}). Observe that, even with a higher innovation rate, most agents (even the incoming ones) still agree on a common lexicon (circled plot). However, they now can emerge a 100% specific lexicon convention ($\sim 50,000$ ticks). In fact, notice that the behavior of spreading with a moderately higher innovation rate on a dynamic environment is very similar to the behavior with a low innovation rate on a static environment. That is to say that innovation plays a key role in dealing with dynamic environments.

Overall, our spreading mechanism is robust enough to deal with dynamic environments where both the agent population and the neighborhoods change with time. However, innovation plays a key role in coping with such dynamic environments.

5.6. Unreliable communications

Our experiments so far have not considered the sensitivity of spreading to unreliable communications. Next, we study how noise affects spreading. We re-ran the experiments in Sections 5.3 and 5.4, but now including noise during information transfers. For each information transfer, we randomly corrupted the transferred lexicons by changing the concept-word associations up to half of the entries (50% of the lexicon) with some probability (p_{noise}) ranging from low to very high ($p_{noise} \in \{0.1, 0.3, 0.6, 0.8\}$). Moreover, we employed the maximum spreading rate (1.0) to study the worst case scenario (the more information transfers sent, the larger the number of corrupted lexicons at any time).

Figure 10 shows the results of our experiments. Observe that even though a convention can still emerge,

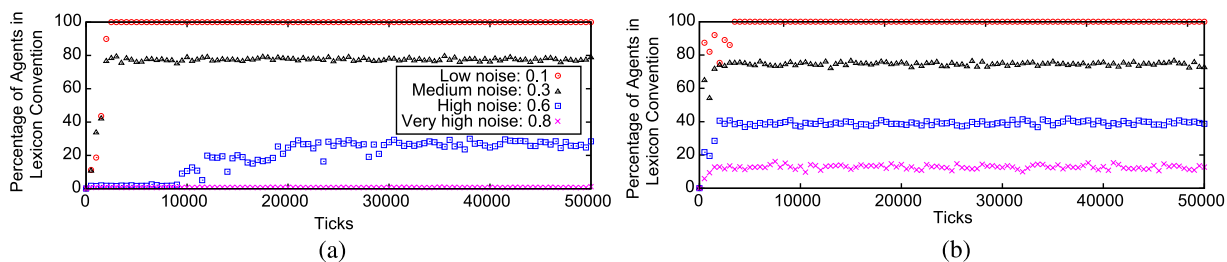


Fig. 10. The effects of different noise levels on emergence. (a) Small-world, (b) scale-free. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

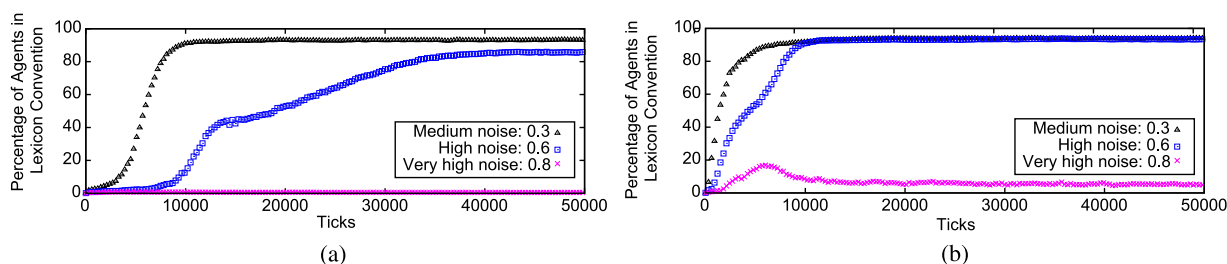


Fig. 11. Convention emergence exploiting self-protection. (a) Small-world, (b) scale-free. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

the higher the noise level the less the number of agents agreeing on a convention. For instance, when the noise level is low agents reach a near global convention. However, as the noise level increases, the number of agents capable of reaching a convention decreases for both topologies.

In scale-free topologies the relationship between the noise level and the number of agents in a shared convention is more visible. For instance, with a high noise level (60% chance of sending corrupted lexicons), it is difficult for more than 40% of the agents to agree on a single convention. However, the emerged conventions are more stable than the ones reached on a small-world topology. This stable behavior is in line with a reported property of scale-free networks, namely their robustness against undirected corruption (e.g., random attacks, viruses) [11], where undirected means that the network hubs are not consciously targeted.

Moreover, noise has another interesting effect, it allows some agents to emerge 100% specific lexicon conventions. However, this is only useful in the low level noise case, since its the only case where most agents reach the lexicon convention. Such effect is consistent with the observation that sometimes a small amount of noise is needed to introduce diversity (the idea behind the innovation component).

To summarize, spreading can only deal with low noise level scenarios. Hence, we need to extend the

spreading mechanism with a further component that protects agents against the higher noise levels.

5.7. The robust spreading mechanism

We observed above that noise is an uncontrollable factor that damages the spreading by degrading the convention seeds that each agent receives. Thus, for a spreading mechanism to be truly *robust* it has to be capable of withstanding, or at least minimizing, the harmful effects of noise. To that end, the experiments in this section aim to empirically validate the effectiveness of the self-protection component (presented in Section 4.4) in both static and dynamic scenarios.

Static. Figure 11 depicts the results when agents employ self-protection (using a window of 30 spreadings) over a static scenario. In general, observe that the complete spreading mechanism (employing the four components) is *robust* because it helps agents emerge the best convention in scenarios with medium and high noise levels over both topologies. In particular, the results are quite significant for high noise levels. Now 90% of the agents agree on a 100% specific lexicon convention, which is significantly better than 40% agreeing on a convention when disregarding self-protection. In other words, a spreading mechanism with self-protection is powerful enough to emerge a top quality convention even when, at any point in time,

Table 2
Convention emergence on various scale-free scenarios

		Static		Dynamic	
		Noiseless	Noisy	Noiseless	Noisy
Convention compliance	Percentage	~100	~90%	~100	~87%
	Time to	Very fast	Very fast	Fast	Fast
Convention quality	Percentage	100%	100%	100%	100%
	Time to	[slow . . . fast] subject to innovation rate	Fast	Slow	Fast

Notes: Compliance – agents in dominant convention. Quality – quality of the highest convention reached. Time to – time it takes to reach compliance and quality.

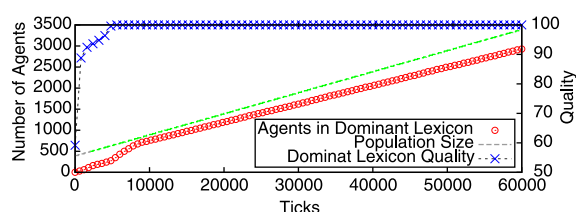


Fig. 12. Results of dynamic agent population with high noise levels. (Colors are visible in the online version of the article; <http://dx.doi.org/10.3233/AIC-2010-0479>.)

there is a high probability that agents propagate unreliable information. Moreover, the resilient behavior of the spreading mechanism is similar for scale-free and small-world topologies, though scale-free neighborhoods reach near global conventions at a faster pace. Regarding scenarios with very high noise levels, it is no surprise that a relevant convention did not emerge since this is an extreme situation.

Dynamic. Figure 12 depicts the results of our experiments when agents continuously join the MAS while the communications are affected by a high noise level. Observe that even in such complex/difficult scenario our spreading mechanism still can reach a 100% specific lexicon convention for ~87% of the agent population. The fact that less agents join the dominant convention (with respect to the noiseless experiment) was expected in a high noise level environment. Specially, since the constant influx of agents with random lexicons also acts as extra noise. In other words, such high agent convention compliance is a significant result. Moreover, a maximum quality lexicon convention is found much faster than in the experiments considering a dynamic population but disregarding noise, reported in Section 5.5 (~6000 ticks against the ~48,000 ticks). This can be explained by the constant exploration resulting from the high noise level.

Overall, the proposed spreading mechanism has shown to be highly robust since it helps agents reach

a perfect communication system for most of the MAS population despite continuous changes in the dynamic population and in the presence of high noise levels.

6. Conclusions and future work

The main contribution of this paper is a robust mechanism for convention emergence that facilitates coordination in multi-agent systems. Overall, our empirical results showed that a spreading mechanism that endows each agent with partial information-transfer, self-protection, elitist selection and innovation (as a sequence of operations): (i) can cope with *a space of multiple convention alternatives*; (ii) functions in dynamic environments; and (iii) is *resilient to unreliable propagations*. So far, no convention emergence mechanism in the literature has addressed these issues. Table 2 summarizes the performance of our mechanism by showing the time it takes (time to) for a percentage of the agent population (percentage) to agree on any convention (convention compliance) and on a top quality convention (convention quality).

The results show that our spreading mechanism can emerge (near-)global conventions regardless of the topology in a largely populated multi-agent system. Nevertheless, the topology does affect the time it takes to reach such convention. On the one hand, agents on a small-world topology smoothly emerge top quality (best) conventions in a very reasonable amount of time. On the other hand, agents on scale-free topologies very rapidly agree to a common, *lower quality*, convention and require more time to find a top quality one.

Furthermore, the spreading and innovation rates play an important role in the behavior of the spreading mechanism. They can improve the effectiveness of spreading and speed up the convergence time to the best convention, specially on scale-free topologies.

In fact, to promptly emerge a top quality global convention (moderately) higher spreading and innovation rates are required, though their values must *counter-balance* each other. For instance, only increasing the innovation rate speeds up the convergence to the best convention but decreases the number of agents that join it.

Additionally, our experiments confirmed that uncontrollable medium or high degrees of noise are very detrimental to traditional spreading. However, our spreading mechanism endowed with self-protection proved to be resilient to such unfavorable conditions (not surprisingly very high noise levels are outside our scope). Not only that, we showed that our mechanism can even emerge conventions on continuously changing (dynamic) MAS that are also affected by high noise levels (although for a slightly lower percentage of the population). This result shows the power of our approach in the worst conditions.

To summarize, although the proposed spreading mechanism has proved to be very powerful, it is not perfect. Firstly, it requires the adequate tuning of some parameters to reach its full potential. Secondly, it cannot cope with the presence of very high noise levels. Nevertheless, observe that the spreading mechanism components admit alternative versions. Therefore, a wealth of convention mechanisms of varying features results from combining such versions. Hence, for future work we plan to attempt to increase our approach robustness by exploring the construction of alternative versions of the self-protection and innovation components capable of parameter self-tuning.

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